Competition between Nonprofit and For-Profit Firms

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Abstract

This paper considers a nonprofit firm competing against a for-profit firm in a homogeneous goods market. Given a stochastic demand function and an asymmetric tax schedule, we derive Cournot-Nash equilibrium allowing the nonprofit firm to have an altruistic preference toward consumer surplus or total surplus. The effects of the tax rate and the degree of altruistic preference on market equilibrium outcomes are analyzed thereof.

Key words: nonprofit; asymmetric taxation; stochastic demand; Cournot-Nash equilibrium

JEL classification: L2; L3

1. Introduction

In some industries, such as education and health care, both nonprofit and for-profit firms coexist and compete against each other. A for-profit firm is subject to profit taxation whereas a nonprofit firm receives benefits from tax exemption [for an estimate of the tax benefits accrued to a nonprofit hospital, see Gentry and Penrod (1998)]. However, nonprofit firms are not allowed to raise capital through equity financing. They also face non-distribution constraints. In the United States, tax practitioners from public accounting firms decide whether a nonprofit hospital should maintain tax exemptions. Reported profits and charitable care are the two major determinants of the final decision [Wilkicki (2001)]. Due to competition and expected future profits, some nonprofit hospitals were converted to for-profit hospitals [Cutler and Horwitz (1998)].

Traditionally, one presumes that a nonprofit hospital obtains tax exemption in pursuit of social objectives [Newhouse (1970)]. Pauly and Redisch (1973) suggest an alternative “sales maximization” type objective function. Recent studies, attributed to the separation of ownership and control, indicate that the distinction between for-profit and nonprofit hospitals is not clear cut. Eldendburg et al. (1999) find financial performance to be the most significant variable in explaining the turnover of nonprofit hospital CEOs. Leone and Van Horn (1999) find nonprofit hospital CEOs
engage in earnings management by adjusting earning figures upward or downward toward a target just above zero. Their results support the importance of financial performance in nonprofit hospitals leading to the CEOs’ earnings management. Barber, Daniel, and Roberts (1999) find changes in top managers pay in nonprofit organizations is related to changes in direct revenue to the organization’s philanthropic objective. Brickley and Van Horn (2000) find the relation between financial performance and CEO turnover and compensation in nonprofit hospitals is as strong as that in for-profit hospitals. Overall, there is strong documented evidence that decision-making in nonprofit hospitals are affected by financial performance (including profits). Weisbrod (1998) argues nonprofit hospitals are more prone to concern for financial performance facing increased competition from for-profit hospitals. Duggan (2000) and Silverman and Skinner (2001) provide empirical support for the conjecture.

In this paper, we consider competition between a nonprofit hospital and a for-profit hospital. It is therefore reasonable to include profit as one of the objectives for the nonprofit hospital. Of course, there are other (altruistic) objectives that the hospital has to account for to justify its nonprofit status. Sansing (2000) constructs an analytical model for competition between a nonprofit firm and a for-profit firm taking into account different production objective functions and different tax treatment. The model is then applied to evaluate the potential of joint ventures between the two firms.

Specifically, Sansing (2000) assumes a linear profit tax with symmetry between gains and losses. If a firm earns profits, it incurs a tax proportional to its profits. On the other hand, if there is a loss, the firm receives a subsidy (i.e., a negative income tax). He also assumes that the objective of the nonprofit hospital can be written as a weighted sum of its own profit and consumer surplus. The Cournot-Nash equilibrium is then derived. The symmetric taxation assumption, however, eliminates the effects of the tax rate on the firm’s production decision. Consequently, the equilibrium is independent of the tax rate despite that tax being recognized as an important element in the model and the key benefit of nonprofit status.

In reality, the tax system is asymmetric such that the firm pays no tax and receives no subsidies when incurring losses. (We ignore carry-backs and carry-forwards. Both allowances serve as subsidies. Nonetheless, the tax schedule remains asymmetric, as neither provides full compensation of losses.) The Sansing assumption is acceptable only when for-profit hospitals rarely incur losses. Using Form 990 tax return data during the 1994-1996 period, Smith (2002) documents that, for a sample of 84 hospitals, the return on assets averaged –0.62% and the return on equity averaged –0.06%. If we restrict ourselves to the 61 for-profit hospitals, the two return averages are –1.47% and –1.18%, respectively. Clearly, for-profit hospitals are likely to incur losses and the symmetric treatment by Sansing (2000) is invalid.

To rectify the problems, this paper incorporates the asymmetric nature of profit tax and a stochastic demand schedule into the Sansing framework. Specifically, instead of a subsidy, there is no subsidy or tax when a for-profit firm incurs a loss. We
construct the Cournot-Nash equilibrium and demonstrate the effect of profit tax on the market equilibrium. Another issue is related to the choice of the altruistic objective for the nonprofit hospital. While Sansing (2000) adopts the consumer surplus, Rhoades-Catanach (2000) suggests the total surplus (i.e., the sum of producer surplus and consumer surplus) be more appropriate. Total surplus is a measure of social welfare and is widely adopted in regulation literature [see, for example, Train (1991)]. As we recognize the pros and cons of the latter objective, we consider both approaches and compare equilibrium outcomes across the two cases.

The paper is organized as follows. In the next section, we describe the benchmark model, i.e., the Sansing (2000) framework. It shows the irrelevance of the tax rate in determining the Cournot-Nash equilibrium. Section 3 incorporates demand uncertainty into the asymmetric taxation framework and presents the modified objective functions for each firm. Optimal production decisions and the resulting Cournot-Nash equilibrium for the modified framework are provided in Section 4. The effects of the tax rate and the nonprofit firm’s altruistic preference (i.e., the weight assigned to the consumer surplus) are discussed in Section 5 through comparative static analysis. Section 6 adopts the Rhoades-Catanach suggestion, replacing consumer surplus with total surplus as the nonprofit firm’s altruistic objective and performs a similar analysis. The results are compared to those derived from the Sansing framework. Finally, Section 7 offers some concluding remarks.

2. The Benchmark Model

The section follows Sansing (2000) closely. Consider two firms competing in a homogeneous product market. Firm 1 is for profit and Firm 2 is nonprofit. The inverse demand function is specified as follows:

\[ p = d - q, \]

where \( p \) is the market price and \( q \) is the quantity demanded. Note that the slope of the inverse demand function is assumed to be \(-1\). This assumption imposes no restriction on linear demand, as we can choose an appropriate measurement unit to satisfy (1).

Suppose that Firm \( j \) has a linear production technique such that the unit production cost is fixed at \( c_j, j = 1, 2 \). It is assumed that \( d > \max\{c_1, c_2\} \) to ensure the firm has a chance to earn profits. Let \( q_j \) denote the production level of Firm \( j \). The profit for Firm \( j \) is then

\[ \pi_j = (p - c_j)q_j = [d - q_1 - q_2 - c_j]q_j. \]

Firm 1 is for profit and its profit is subject to taxation. Let \( t \) denote the profit tax rate. The firm incurs a tax of \( t\pi_1 \) (\( 0 < t < 1 \)). Note that this tax structure provides the firm a subsidy when it incurs a loss. The after-tax profit is then \( \pi_1^t = (1-t)\pi_1 \). Firm 1 chooses the optimal production level to maximize its after-tax profit.
ever, maximizing $\pi_1^*$ is equivalent to maximizing $\pi_1$. The tax rate has no effect on the firm’s production decision.

On the other hand, Firm 2 is a nonprofit organization. Therefore, in choosing the optimal production level, it must take into account an altruistic objective. Sansing (2000) adopts the consumer surplus in his study. Due to its not-for-profit nature, Firm 2 is not subject to any profit taxation. For a given demand quantity, $q$, the consumer surplus is measured as follows:

$$CS = \int_0^q (d - z)dz - (d - q)q = (1/2)q^2.$$ (3)

Let $w$ denote the importance of the consumer surplus for Firm 2 relative to its profit. The firm’s objective function (or utility function) is written as $V = \pi_2 + w \cdot CS$, or

$$V = [d - q_1 - q_2 - c_2]q_2 + (w/2)(q_1 + q_2)^2,$$ (4)

where $w$ is the weight assigned to the consumer surplus (i.e., the firm’s altruistic preference). Firm 2 chooses its optimal production level to maximize $V$.

We consider a Cournot-Nash equilibrium in which each firm chooses its optimal production level assuming the other firm maintains its current production level. To maximize $\pi_1^*$, the optimal production level, $q_1^*$, must satisfy the following first-order condition:

$$d - 2q_1 - q_2 - c_1 = 0,$$ (5)

whereas the second order condition obviously holds. Similarly, to maximize $V$, the optimal production level of Firm 2, $q_2^*$, must satisfy the following equation:

$$\frac{\partial V}{\partial q_2} = d - q_1 - 2q_2 - c_2 + w(q_1 + q_2) = 0.$$ (6)

The second-order condition requires $w \leq 2$. That is, the nonprofit firm cannot weight the consumer surplus more than twice of its own profit. Although peculiar, this assumption is needed to derive the market equilibrium for the Sansing setup. By solving equations (5) and (6) simultaneously, we have

$$q_1^* = \frac{(1 - w)d + c_1 - (2 - w)c_1}{3 - w}$$ (7a)

$$q_2^* = \frac{(1 + w)d - 2c_2 + (1 - w)c_1}{3 - w}.$$ (7b)

Moreover $\frac{\partial q_1^*}{\partial w} = (2d - c_1 + c_2) / (3 - w)^2 < 0$ and $\frac{\partial q_2^*}{\partial w} = -2(\frac{\partial q_1^*}{\partial w}) > 0$. As the nonprofit firm becomes more altruistic, it expands the production level to enhance consumer surplus.
3. Demand Uncertainty and Asymmetric Taxation

Suppose that we replace the symmetric taxation with a more realistic asymmetric taxation in the framework such that the firm incurs a tax of \( \pi_1 (0 < t < 1) \) only if it earns a profit, i.e., \( \pi_1 > 0 \). No tax is imposed and no subsidy is provided when the firm incurs a loss, i.e., \( \pi_1 < 0 \). The after-tax profit is then

\[
\pi_1^a = \pi_1 - t \max \{\pi_1, 0\}.
\] (8)

Firm 1 chooses the optimal production level to maximize its after-tax profit. Within the current demand and cost structure, the firm will not incur any loss. Consequently, \( \pi_1^a = (1 - t) \pi_1 \). Maximizing \( \pi_1^a \) is equivalent to maximizing \( \pi_1 \). Once again, the tax rate has no effect on the firm’s production decision.

To highlight the effect of the tax rate, either the demand function or the cost function must be modified. In this paper, we consider a stochastic inverse demand function such that

\[
\epsilon qdp,
\] (9)

where \( \epsilon \) is a zero-mean random variable representing the demand shock. Both firms make their production decisions prior to the realization of the demand shock.

Given the stochastic demand, the profit for Firm \( j \) is also stochastic such that

\[
\pi_j = (p - c_j)q_j = [d - q_1 - q_2 + \epsilon - c_j]q_j.
\] (10)

Under asymmetric taxation, the after-tax profit for Firm 1 is described in (8). Firm 1 chooses the optimal production level to maximize its expected profit, \( E(\pi_1^a) \), where the expectation is taken over \( \epsilon \). Note that \( \pi_1 > 0 \) is equivalent to the condition that \( p > c_1 \) or \( \epsilon > c_1 + q_1 + q_2 - d \). Let \( f(\cdot) \) denote the probability density function of \( \epsilon \). We have the following:

\[
E(\pi_1^a) = \int_{\pi_1^a} \left[ \int_{\pi_1}^{\infty} [d - q_1 - q_2 - c_1 + \epsilon] f(\epsilon) d\epsilon \right] d\pi_1.
\] (11)

On the other hand, Firm 2 attempts to maximize the expectation of a weighted sum of its own profit and consumer surplus. For a given demand quantity, \( q \), the expected consumer surplus is:

\[
E(CS) = E \left[ \int_0^q (d - z + \epsilon) dz - (d - q + \epsilon)q \right] = (1/2)q^2.
\] (12)

As a result, the objective function of Firm 2 is \( EV = E(\pi_2) + wE(CS) \), or

\[
EV = [d - q_1 - q_2 - c_2]q_2 + (w/2)(q_1 + q_2)^2.
\] (13)
Firm 2 chooses its optimal production level to maximize \( EV \). Note that \( EV \) is the same as \( V \). Thus, the addition of demand shock directly affects the production behavior of Firm 1. It has no direct impact on Firm 2. That is, for a given Firm 1 output level, Firm 2’s production is the same with or without demand shock. However, as demand shock induces Firm 1 to changes its production level, strategic interactions between the two firms then lead Firm 2 to adjust its production level as well.

4. Production Decisions and Market Equilibrium

We now characterize each firm’s production decision and the resulting Cournot-Nash equilibrium. To maximize \( E(\pi^1) \), the optimal production level, \( q_1^* \), must solve the following equation:

\[
\frac{\partial E(\pi^1)}{\partial q_1} = d - 2q_1 + q_2 - c_1 - \tau \int (d - 2q_1 - q_2 - c_1 + c) f(c) dc = 0.
\] (14)

Let \( \tau = t/(1-t) \) and let

\[
H(z) = \int \frac{c}{z} f(c) dc.
\] (15)

Because \( E(c) = 0 \), \( H(z) \) is always positive. Equation (14) can be rewritten as follows:

\[
d - 2q_1 + q_2 - c_1 - \tau H(c_1 + q_1 + q_2 - d) = 0.
\] (16)

Note that, when \( \tau = 0 \), the above equation is reduced to the case in Sansing (2000). This is not surprising, as the profit tax has no impact within the Sansing framework.

From (17), \( d - q_1 - q_2 - c_1 = q_1 + \tau H(c_1 + q_1 + q_2 - d) > 0 \). Using the result \( dH(z) / dz = -zf(z) \), we have

\[
\frac{\partial^2 E(\pi^1)}{\partial q_1^2} = -2(1-t) + \tau(c_1 + q_1 + q_2 - d) f(c_1 + q_1 + q_2 - d) < 0.
\] (17)

Thus, the second-order condition for maximization is always satisfied when (16) holds.

To maximize \( EV \), the optimal production level of Firm 2, \( q_2^* \), must satisfy the following equation:

\[
\frac{\partial EV}{\partial q_2} = d - q_1 - 2q_2 - c_2 + w(q_1 + q_2) = 0.
\] (18)
The second-order condition of maximization requires \( \frac{\partial^2 EV}{\partial q_1^2} < 0 \), which in turn requires \( w < 2 \). The firm cannot weight the consumer surplus more than twice of its own profit.

The Cournot-Nash equilibrium is derived by solving (16) and (18) simultaneously. From (18),

\[ q_2 = \left(\frac{1}{2-w}\right)[d - (1-w)q_1 - c_2]. \]  

(19)

Substituting the relationship into (16) and after algebraic manipulation, we derive

\[ (d - q_1 - c_1) + \left(\frac{1}{2-w}\right)(c_2 - d - q_1) = \tau (c_1 - d) + \left(\frac{1}{2-w}\right)(d + q_1 - c_2). \]  

(20)

The solution to the above equation provides \( q_1^* \). Substitution of this value into (19) derives \( q_2^* \). When \( \tau = 0 \), the Cournot-Nash equilibrium solution coincides with that in Sansing (2000).

5. Comparative Static Analysis

In this section, we evaluate the effects of the tax rate and the preference of consumer surplus on the market equilibrium. Let \( m = (c_1 - d) + (2-w)^{-1}(d + q_1 - c_2) \). Applying comparative static analysis, we have

\[ \frac{\partial q_1^*}{\partial \tau} = -H(m)\left[1 + \left(\frac{1}{2-w}\right)^{-\tau} mf(m)^2 \left(\frac{1}{2-w}\right)^{-1}\right]. \]  

(21)

From (20), \( m = -q_1 - \tau H(m) < 0 \) because \( H(m) > 0 \). Thus \( \frac{\partial q_1^*}{\partial \tau} < 0 \), implying \( \frac{\partial q_1^*}{\partial \tau} < 0 \). Firm 1 produces less when the profit tax rate increases. Note that Sansing’s results correspond to the case of a zero tax rate. We conclude that the recognition of asymmetric tax treatment leads to a smaller output level for the for-profit firm.

Turning to Firm 2, from (19) we derive

\[ \frac{\partial q_2^*}{\partial \tau} = \left(\frac{\partial q_1^*}{\partial \tau}\right) \left(\frac{\partial q_2^*}{\partial q_1}\right) = \left(\frac{w-1}{2-w}\right) \left(\frac{\partial q_1^*}{\partial \tau}\right). \]  

(22)

Thus, \( \frac{\partial q_2^*}{\partial \tau} < 0 \) if \( w > 1 \) and \( \frac{\partial q_2^*}{\partial \tau} > 0 \) if \( w < 1 \). As the profit tax rate increases, the nonprofit firm increases (decreases) its output if the consumer surplus is weighted less (more) than its own profit. On the other hand,

\[ \frac{\partial (q_1^* + q_2^*)}{\partial \tau} = \left(\frac{\partial q_1^*}{\partial \tau}\right) + \left(\frac{w-1}{2-w}\right) \left(\frac{\partial q_1^*}{\partial \tau}\right) = \left(\frac{1}{2-w}\right) < 0. \]  

(23)
Regardless of Firm 2’s production decision, the total output decreases with an increasing tax rate, leading to an increase in the market price. These results are summarized in the following proposition.

**Proposition 1**: As the profit tax rate increases, the for-profit firm reduces its production whereas the nonprofit firm increases (decreases) its production if it weights consumer surplus less (more) than own profit. The overall effect is a reduction in the total production and, therefore, an increase in market price.

Before the tax rate increase, at the market equilibrium, Firm 1 produces at the point where marginal revenue equals (the constant) marginal cost. After the increase, marginal revenue decreases and falls short of the marginal cost. As a result, Firm 2 reduces its production level. The marginal utility for Firm 2 (i.e., $\frac{\partial EV}{\partial q_2}$), however, may increase or decrease. On the one hand, the marginal consumer surplus decreases as Firm 1 produces less. On the other hand, the marginal revenue increases due to a reduction in the total output. The two opposing effects are equal to each other in magnitude (because of the linear demand assumption). When the firm weights consumer surplus more than own profit (i.e., $w > 1$), the first effect dominates the second. The marginal utility falls short of the marginal cost. Consequently, Firm 2 produces less. If, instead, the firm weights consumer surplus less (i.e., $w < 1$), then the second effect dominates. The marginal utility exceeds the marginal cost, resulting in an increase in Firm 1’s production level. However, due to the trade-off between the two opposing effects, the expansion of Firm 2 is not sufficient to compensate for the contraction of Firm 1. The total output is therefore decreased after the tax rate increase.

To analyze the effect of the preference for consumer surplus, let $v = 1/(2 - w)$. Applying comparative static analysis to (20),

\[
\frac{\partial q_1^*}{\partial v} = \frac{(d + q_1^* - c_2)[1 - \tau \cdot mf(m)]}{-1 - v + \tau \cdot mf(m)}.
\]  

(24)

Note that $m < 0$ and $v > 0$. Also, from (19),

\[
q_1^* + q_2^* = \left(\frac{1}{2-w}\right)(d + q_1^* - c_2).
\]  

(25)

Because $q_1^* + q_2^* > 0$, we have $d + q_1^* - c_2 > 0$. Hence, $\frac{\partial q_1^*}{\partial v} < 0$ and $\frac{\partial q_2^*}{\partial w} < 0$ due to the fact that $v$ is an increasing function of $w$. When Firm 2 cares more about consumer surplus, Firm 1 reduces its production level. Moreover, (19) implies

\[
\frac{\partial q_2^*}{\partial v} = d - c_2 + q_1^* + (v-1)\left(\frac{\partial q_1^*}{\partial v}\right).
\]  

(26)

Substituting (24) into the above equation and after algebraic manipulation, we
have

$$\frac{\partial q_2^*}{\partial v} = \frac{(d + q_1^* - c_2)[2-\tau \, mf(m)]}{1+\nu - \tau \, vmf(m)} > 0.$$  \hfill (27)

Thus, $\frac{\partial q_2^*}{\partial w} > 0$. Firm 2 expands its production when the consumer surplus becomes more important to its operations. In fact,

$$\frac{\partial (q_1^* + q_2^*)}{\partial v} = \frac{d + q_1^* - c_2}{1+\nu - \tau \, vmf(m)} > 0.$$  \hfill (28)

That is, the increase in Firm 2’s production more than offsets the reduction in Firm 1’s production. Proposition 2 summarizes the above results.

**Proposition 2:** If the nonprofit firm values the consumer surplus higher, it expands its production level. Meanwhile, the for-profit firm produces less. The overall effect is an increase in the total production and, therefore, a reduction in the market price.

An intuitive explanation of the above results is as follows. At the initial market equilibrium, Firm 2 produces at the level where marginal utility equals (the constant) marginal cost. When the altruistic preference increases, marginal utility increases, exceeding the marginal cost. Consequently, Firm 2 expands its production, driving down the market price. The marginal revenue of Firm 1 is then smaller than its marginal cost, which induces the firm to produce less. However, the reduction in the for-profit firm’s production is less than the increase in the nonprofit firm’s production. The total output increases as Firm 2 becomes more altruistic.

### 6. Alternative Objective Function

In the previous sections, we follow Sansing (2000) adopting consumer surplus as the measurement for the nonprofit firm’s altruism. Lakdawalla and Philipson (1998) suggest the nonprofit firm’s own production level as the altruism indicator. Rhoades-Catanach (2000) argues for the total surplus (i.e., the sum of consumer surplus and producer surplus); an approach echoed in the regulation literature [see, for example, Train (1991)]. Total surplus is well accepted as a measure of social welfare and seems appropriate to serve as an altruism indicator. We therefore adopt this method in the current section. It is worthwhile pointing out that, as total surplus includes the for-profit firm’s profit, we assume Firm 2 cares about Firm 1’s profit level. Also, as total surplus includes Firm 2’s own profit, there may be a concern for double counting. This concern, however, can be addressed as follows. Assuming a linear, additive specification, the most important factor in Firm 2’s objective function is the weight allocation to the three arguments: its own profit, Firm 1’s profit, and the consumer surplus. Sansing (2000) chooses $1/(1+w)$, 0, and $w/(1+w)$, respectively. Using total surplus as altruism indicator, the weights assigned are
(1 + w)/(1 + 3w), w/(1 + 3w), and w/(1 + 3w), respectively. The latter approach merely gives the firm’s own profit a greater weight than the other two arguments. It also gives Firm 1’s profit an equal weight to the consumer surplus.

The producer surplus is calculated as follows:

\[ PS = (d - q_1 - q_2 + \epsilon - c_1)q_1 + (d - q_1 - q_2 + \epsilon - c_2)q_2 . \]  \hspace{1cm} (29)

As a result, the expected total surplus is \( E(TS) = E(CS) + E(PS) \), or

\[ E(TS) = (1/2)(q_1 + q_2)^2 + (d - q_1 - q_2)(q_1 + q_2) - c_1q_1 - c_2q_2 . \]  \hspace{1cm} (30)

Firm 2 chooses its production level to maximize \( EW = E(\sigma_2) + wE(TS) \). The optimal production level, \( q^*_2 \), satisfies the following equation:

\[ \frac{\partial EW}{\partial q_2} = (1 + w) \left[ d - q_1 - c_2 - \left( \frac{2 + w}{1 + w} \right) q_2 \right] = 0 . \]  \hspace{1cm} (31)

The second-order condition, \( \frac{\partial^2 EW}{\partial q^2} < 0 \), is always satisfied regardless of the value of \( w \), a desirable departure from the Sansing case.

The Cournot-Nash equilibrium is derived by solving (16) and (31) simultaneously. From (31),

\[ q_1 = d - c_2 - \left( \frac{2 + w}{1 + w} \right) q_2 . \]  \hspace{1cm} (32)

Substituting this relationship into (16), we have

\[ -d + c_1 + 2c_2 + \left( \frac{3 + w}{1 + w} \right) q_2 = \tau H \left( c_1 - c_2 - \frac{q_2}{1 + w} \right) . \]  \hspace{1cm} (33)

By comparative static analysis,

\[ \frac{\partial q^*_2}{\partial \tau} = \frac{(1 + w) H(n)}{3 + w - \tau n f(n)} , \]  \hspace{1cm} (34)

where \( n = c_1 - c_2 - (1 + w)^{-1} q_2 \). The next lemma is useful.

**Lemma 1:** At the market equilibrium, \( n < 0 \).

**Proof 1:** From (33),

\[ -d + c_1 + 2c_2 + \left( \frac{3 + w}{1 + w} \right) q_2 > 0 , \]

which can be rewritten as
Substituting (32), we have
\[-\left(c_1 - c_2 - \frac{q_2}{1+w}\right) - d + c_2 + \left(\frac{2+w}{1+w}\right)q_2 > 0,\]

which implies
\[n = \left(c_1 - c_2 - \frac{q_2}{1+w}\right) < -q_1 < 0.\]

As a consequence, (32) implies \(\partial q_2^f / \partial \tau > 0\) and hence \(\partial q_2^f / \partial \tau > 0\). Moreover, (32) leads to
\[\frac{\partial q_2^f}{\partial \tau} = -\left(\frac{2+w}{1+w}\right)\left(\frac{\partial q_2^f}{\partial \tau}\right) < 0,\] (35)
or \(\partial q_2^f / \partial \tau < 0\). Similarly, we have
\[\frac{\partial(q_1^f + q_2^f)}{\partial \tau} = -\left(\frac{1}{1+w}\right)\left(\frac{\partial q_2^f}{\partial \tau}\right) < 0,\] (36)
or \(\partial(q_1^f + q_2^f) / \partial \tau < 0\). Proposition 3 summarizes the results.

**Proposition 3**: As the profit tax rate increases, the for-profit firm reduces its production whereas the nonprofit firm increases its production. However, the increase is less than the reduction. As a result, the total production decreases, leading to a higher market price.

Proposition 3 is similar to Proposition 1. Therefore, the intuition provided for Proposition 1 also applies. The main difference lies in the following result. When the nonprofit firm cares for consumer surplus, in response to a tax rate increase it may increase or decrease the production level depending upon the degree of the altruism preference. In case of the total surplus, there are two additional terms in the altruism indicator, Firm 2’s own profit and Firm 1’s profit. As the increase in Firm 2’s marginal profit exceeds the reduction in Firm 1’s marginal profit, it helps compensate some of the losses in marginal consumer surplus. Consequently, the nonprofit firm always increases its production as the tax rate increases, regardless of the degree of altruism preference.

To evaluate the effect of Firm 2’s preference for total surplus, let \(k = (1+w)^{-1}\). Equation (33) can rewritten as:
\[-d + c_1 + 2c_2 + (1+2k)q_2 = \tau (c_1 - c_2 - kq_2),\] (37)
from which we derive
\[
\frac{\partial q_k^x}{\partial k} = \frac{-2 + \tau n f(n)q_k^x}{1 + 2k - \tau knf(n)} < 0 ,
\] (38)
because \( n < 0 \). On the other hand, (32) shows \( q_1 = d - c_2 - (1+k)q_2 \). After algebraic manipulation, we have
\[
\frac{\partial q_k^x}{\partial k} = -q_k^x - (1+k)\left(\frac{\partial q_k^x}{\partial k}\right) + \frac{1 - \tau n f(n)q_k^x}{1 + 2k - \tau knf(n)} > 0 ,
\] (39)
\[
\frac{\partial (q_k^x + q_k^y)}{\partial k} = \frac{-q_k^x}{1 + 2k - \tau knf(n)} < 0 .
\] (40)

Note that \( k \) is a decreasing function of \( w \). The above results imply \( \frac{\partial q_k^x}{\partial w} > 0 , \frac{\partial q_k^x}{\partial w} < 0 \), and \( \frac{\partial (q_k^x + q_k^y)}{\partial w} > 0 \).

**Proposition 4**: When the nonprofit firm places a greater weight on the total surplus (i.e., social welfare), it expands its production level. Meanwhile, the for-profit firm produces less. The overall effect is an increase in the total production and, therefore, a reduction in the market price.

The above results are identical to those in Proposition 2. The intuition provided there is directly applicable.

Overall, the Rhoades-Catanach objective function creates two departures from the Sansing results. First, there is no need to impose a restriction on the weight for the social welfare to derive the interior solution. Secondly, the nonprofit firm always produces more in response to an increase in the tax rate. Intuitively, when a larger tax is imposed, the for-profit firm produces less. To enhance social welfare, it is expected that the nonprofit firm produces more. Both departures appear to be analytically desirable and intuitive. In other words, although which indicator—consumer surplus or total surplus—is a better description of the nonprofit firm remains an empirical question, analytically we find the results derived under the total surplus criteria are more intuitive and conforming to conventional wisdom.

We now consider two market scenarios. In the first scenario, a for-profit firm competes against a nonprofit firm where the nonprofit firm cares for consumer surplus in addition to its own profit. For the second scenario, the nonprofit firm cares for total surplus in addition to its own profit. When comparing Cournot-Nash equilibria across the two scenarios, we have \( q_1^x > q_1^y , q_2^x < q_2^y \), and \( q_1^x + q_2^x < q_1^y + q_2^y \). The results are summarized in the following proposition.

**Proposition 5**: Suppose we compare two market scenarios. In the first scenario, the nonprofit firm adopts consumer surplus as altruism indicator and in the second scenario it cares for total surplus. The for-profit firm produces more whereas the nonprofit firm produces less in the second scenario, as compared to the first scenario. The total production is larger in the second scenario, leading to a lower market price.
To prove Proposition 5, note that (33) implies

$$q_2 = \left( \frac{1+w}{2+w} \right) (d - q_1 - c_2).$$ \hspace{1cm} (41)

The difference between the right-hand-side of the above equation and the right-hand-side of (18) is

$$\left( -\frac{w^2}{4-w^2} \right) (d - c_2) - \left( \frac{2w}{4-w^2} \right) q_1 < 0.$$

Under the Sansing framework, the market equilibrium is the simultaneous solution to (16) and (19). For the Rhoades-Catanach model, the market equilibrium is the simultaneous solution to (16) and (31). The above inequality shows that the line described by (19) lies above the curve described by (31) in the \((q_1, q_2)\) plane. Moreover, (16) describes \(q_2\) as a decreasing function of \(q_1\) with the derivative

$$\frac{dq_2}{dq_1} = -\frac{2-\theta f(\theta)}{1-\theta f(\theta)} = -\left[ 1 + \frac{1}{1-\theta f(\theta)} \right] < -1,$$

where \(\theta = c_1 + q_1 + q_2 - d < 0\). As a result, the curve described by (16) intersects the line corresponding to (19) before it intersects with the line corresponding to (31), leading to \(q_1^* > q_1^*\) and \(q_2^* < q_2^*\). Because the slope of the curve described by (16) is always less than \(-1\), we conclude \(q_1^* + q_2^* < q_1^* + q_2^*\).

Intuitively, in the second scenario, the profit of Firm 1 is a part of Firm 2’s objective function. When Firm 1 produces more, Firm 2 produces less than it does in the first scenario to help maintain a high profit for Firm 1. Consequently, Firm 1 becomes more aggressive and produces more in the second scenario than it does in the first scenario. Firm 2 responds by reducing its production. However, because Firm 2 also cares for consumer surplus, the reduction is less than the expansion by Firm 1. The total output therefore is larger in the second scenario.

7. Conclusions

This paper begins with the Sansing (2000) framework to analyze the market equilibrium in which a nonprofit firm competes against a for-profit firm. It is shown that the tax rate (the main advantage of nonprofit status) has no effect on the market outcome. To account for the effect of the tax rate, a stochastic demand function and an asymmetric profit tax schedule are incorporated. Suppose that the nonprofit firm adopts consumer surplus as its altruistic objective. Within the new framework, as the tax rate increases, the for-profit firm reduces its production whereas the nonprofit firm may produce more or less depending upon the degree of the altruistic preference. A firm weighting consumer surplus more (less) than its own profit decreases (increases) its production. The total output is always reduced though. On the other hand, if the nonprofit firm cares for total surplus, then it will always produce more
in response to a tax rate increase, regardless of the degree of altruistic preference. Nonetheless, the total output remains smaller after the tax rate increase.

Regardless of the altruism indicator (i.e., either consumer surplus or total surplus), a more altruistic nonprofit firm produces more. In response, the for-profit firm produces less at the market equilibrium. The total output, however, is increased and the equilibrium price decreased.

Finally, we compare two market scenarios. In the first scenario, a for-profit firm competes against a nonprofit firm with consumer surplus as its altruism indicator. In the second, the nonprofit firm is concerned with total surplus. When comparing market equilibria across the two scenarios, we find the for-profit firm produces more and the nonprofit firm produces less in the second scenario. The total output is larger in the second scenario. While which indicator—consumer surplus or total surplus—is a better description for the nonprofit firm remains an empirical question, analytically we find the results derived under the total surplus criteria are more intuitive and conforming to conventional wisdom.

References


