Cost Pass-Through with Network Externalities

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Abstract  
We analyze the rate at which cost shocks are passed through to prices when the market exhibits network externalities. We find that the pass-through rate is smaller in the presence of network externalities. Also, the deadweight loss created by a cost shock is smaller in a network market.

Key words: network externalities; cost pass-through; deadweight loss

JEL classification: D6; D85; H23

1. Introduction  
Cost pass-through is an important theme in several fields of economics, including international economics, industrial organization, antitrust policy, and public economics. International economists have investigated how the variation in imported input prices that result from exchange rate fluctuations are passed through to consumers and why in many cases such cost shocks are not fully reflected in price changes (e.g., Goldberg and Knetter, 1997). The impact of how changes in commodity prices impact the price paid by consumers is of interest to everyone in society; there are many examples of this issue, from how crude oil price changes affect gasoline the prices at the pump (e.g., Bacon, 1991; Borenstein et al., 1997; Johnson, 2002) to how the price of beef at the farm level gets transmitted to the retail price (e.g., Goodwin and Holt, 1999). In addition, cost pass-through is the central concept in the much debated issue of whether efficiency gains can be used as a valid argument in merger cases (Kwoka and White, 2008). Similarly, changes in excise, or per-unit, taxes have a clear resemblance to a cost shock and hence there have been important efforts in the public economics literature to understand issues such as the determinants of the tax pass-through rate and the consequent effectiveness for revenue generation or consumption reduction (Bishop, 1968; Fullerton and Metcalf, 2002). Important examples of this literature are cigarette and

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alcohol taxes at both the state and federal level.

Economists have explored from different angles the theoretical reasons for cost (or per-unit tax) over- and under-shifting with analyses that include demand functional form (Cotterill et al., 2001; Delipalla and Keen, 1992), strategic interaction (Peitz and Resinger, 2009, Anderson et al., 2001), and menu costs and habit formation (Ravn et al., 2008). In this paper we add to this literature by theoretically studying how the pass-through rate can be affected when the market exhibits positive network externalities.

In network markets, consumers’ willingness to pay for the last unit increases as the (expected) number of subscribers escalates, implying that the network experiences a self-sustained growth beyond a minimal number of subscribers (i.e., the critical mass). We build comparative statics in monopoly and oligopoly markets that examine whether the cost pass-through rate is smaller or larger than in a no-network-externality counterpart. The comparative static is based on a cost-side shift which can be viewed as a shock in technology or input prices or, alternatively, the introduction (or increase) of a per-unit tax (also known as an excise tax). In addition, given the important policy and social ramifications of cost pass-through outlined earlier, we complement our analysis with welfare comparisons.

When compared to a no-network-externalities counterpart, we find that the pass-through rate is smaller in a market with network externalities. Specifically, if a tax increase causes the same reduction in quantity in both the network externalities case and its standard counterpart, the price increase observed in the network externalities case is smaller. Intuitively, because consumers dislike a smaller network (as a consequence of the price increase), the price increase the market can bear must be smaller to keep the equilibrium quantity change the same as in the case without network effects. In addition, we also show how the associated market distortion (measured by the welfare loss) created by a cost shock (or a per-unit tax) is smaller in a network market.

2. Literature Review

2.1 Cost Pass-Through

The review provided here is by no means a comprehensive overview of the literature. Instead, it should be viewed as a brief summary of the theoretical and empirical findings regarding cost pass-through that relate most closely to our study.

Relevant Theoretical Results

Throughout this article we adopt the usual definition of pass-through rate (PTR): the effect of a change in marginal cost on the equilibrium price \( \frac{\partial p}{\partial c} \). Using this definition, cost shocks are said to be under-shifted if \( \frac{\partial p}{\partial c} < 1 \), over-shifted if \( \frac{\partial p}{\partial c} > 1 \), and completely passed-through if \( \frac{\partial p}{\partial c} = 1 \). In a perfectly competitive market, cost pass-through can at most be equal to 1, with the magnitude depending on the demand and supply elasticities (Fullerton and Metcalf, 2002). At
the other extreme, monopoly, cost shocks can be under- or over-shifted: under linear demand and linear supply the pass-through rate can at most be 0.50, whereas under a constant-elasticity demand and constant marginal costs it will be greater than 1 (Harris and Sullivan, 1979; Delipalla and Keen, 1992; Besley, 1998; Cotterill et al., 2001; Fullerton and Metcalf, 2002).

As in monopoly, oligopoly markets can experience under- or over-shifting, albeit for a different reason. When strategic interaction is important and there is a common cost shock, firms react because they face higher costs but also because their rivals accommodate to the new environment. In Bertrand competition with differentiated products, upward sloping reaction functions trigger a multiplicative effect of the cost shock that propagates throughout the industry and can result in over-shifting (Anderson et al., 2001). In Cournot oligopoly with homogeneous products, a positive cost shock allows firms to escape the prisoners’ dilemma by making every firm credibly commit to smaller levels of production (Seade, 1987). Successive market power in vertically separated markets can also exacerbate the pass-through rate; the reason for this is that a cost shock that occurs at the top of the supply chain is subject to more than one mark-up (Peitz and Resinger, 2009).

Multiple estimates have shown consistent under-shifting of exchange rates (e.g., Engel, 1999; Parsley and Wei, 2001; Goldberg and Campa, 2006). One theoretical explanation has to do with local costs: if imported inputs are not too important in the production process, domestic prices should not respond too much to exchange rate fluctuations (Sanyal and Jones, 1982; Burstein et al., 2003; Corsetti and Dedola, 2004). Another has to do with price rigidity resulting from firms’ dynamic considerations: producers set a price taking into account both current and future costs as well as demand (Devereux and Engel, 2001; Ravn et al., 2008).

Related Empirical Findings

While the cost pass-through can occur as a result of any typical cost change, such as technology, weather, wage increases, or any other change in input prices, research on cost pass-through has been mainly motivated and studied around specific cost change scenarios: mergers, taxes, and exchange rates. While there is a large empirical literature documenting pass-through rates, there appear to be two important sets of (a priori) “at odds” findings. On the one hand, the international economics literature suggests that under-shifting is a consistent and well documented finding (i.e., over-shifting does not occur). On the other hand, the public economics and industrial organization literatures tend to suggest that over-shifting is a common phenomenon.

On the public economics/industrial organization front, there have been several studies documenting the PTR. Poterba (1999) reviews a number of studies that document over-shifting. Besley and Rosen (1998) also find evidence for over-shifting in several industries, although Poterba’s own analysis yields mixed findings. Barzel (1976) documents PTR over-shifting in the cigarette industry while Hanson and Sullivan (2008) confirm this finding with more detailed data. There have also been structural efforts either documenting or explaining PTR over-shifting (Kim and
Cotterill, 2008; Rojas, 2008).

On the international economics front, there are extensive reviews that document and explain PTE under-shifting (Menon, 1995; Goldberg and Knetter, 1997). More recent efforts have taken a structural approach (e.g., Goldberg and Hellerstein, 2008; Nakamura and Zerom, 2010); these last two studies find that local costs explain a large portion of the under-shifting pattern.

2.2 Network Externalities

When utility and/or firms’ profits are directly affected by the number of consumers and/or producers using the same (or a compatible) technology, this technology is said to exhibit network externalities (Church and Gangal, 1993; Farrel and Soloner, 1985; Katz and Shapiro, 1985, 1986; Shapiro and Varian, 1999); consumers of such a technology belong to a network. The fundamental idea is that the act of joining a network confers a benefit to all other participants in the network. Therefore, network externalities may affect consumers’ decisions whether to adopt a new technology and/or producers’ decisions whether to standardize their products. Communication technologies are the main example of network goods. Technologies that have strong network externalities generally live longer and have a rapid growth after passing a critical dimension. This is the consequence of a positive feedback that network industries exhibit: when the subscriber’s base of a network grows, an increasing number of users will find it more profitable to join. A natural consequence in a market with a strong positive feedback is that fewer firms (than otherwise) emerge thereby resulting in increased market dominance.

The literature on network effects usually distinguishes among two types of network externalities: direct network externalities and indirect network externalities. With direct network externalities the utility of an agent connected to the network good positively depends on the number of other users consuming the same good. The telephone market (both fixed and mobile) as well as the fax market exhibit direct network externalities. Subscription to a mobile network, for example, directly benefits existing users of such network, who now have an additional person with whom they may communicate. If there are \( n \) consumers connected to the network, each user can potentially communicate with \( n(n-1) \) users; an additional consumer therefore adds \( 2n \) (total) potential communications within the system, and thus enhances the value of membership, assuming that each user may at some point wish to communicate with every other owner (Economides, 1996).

With indirect network externalities, the value of a good increases as the number, or variety, of complementary goods increases: the addition of new varieties of one type of components affects positively but indirectly the utility of all participants through the reduction of prices. Generally, most markets with indirect network externalities are divided into two distinct sides which benefit from the interaction between them. Typical examples are the personal computer market and credit card networks. Katz and Shapiro (1994) illustrate this type of system with the hardware/software paradigm, which can be extended to many other product combinations such as cameras and film, phonographs and records, and television
sets and programming.

Technically, the source of network externalities lies in consumers’ expectations about the future installed base of subscribers. For example, the demand for a fax machine is a function not only of the price of the product, but also of the expected size of the network to which the fax machine will be connected. This implies that with network externalities the fundamental relationship between price and quantity may fail. For these goods, the willingness to pay for the last unit may increase as the number of subscribers that are expected to connect to the network raises. Thus the (fulfilled expectations) demand-price schedule may not slope downward everywhere but it can show an upward sloping portion; as costs decrease one may observe discontinuous expansions in sales rather than the smooth expansion along a downward sloping demand curve. This occurs because, if the number of people who connect to the network is low, then the marginal individual willingness to pay is low (there are not many other people to communicate with); if there is a large number of people connected, then the willingness of the marginal individual to pay also is low (everyone else who has valued it higher is already connected) (Economides, 1996; Economides and Himmelberg, 1995).

Figure 1. Demand Function with Network Externalities

![Diagram of demand function with network externalities](image.jpg)

Figure 1 shows an illustration of the shape of the demand curve just described. The network starts at essentially zero, with a few small perturbations over time. As cost decreases (due, for example, to technological progress) the network reaches a critical mass (the unstable equilibrium) that kicks up past the low-level equilibrium and the system then reaches the high-level equilibrium (as shown by the arrows in Figure 1). Therefore the two stable equilibria are: (a) zero subscribers or (b) the highest possible level of the network size (i.e., where cost intersects the downward sloping section of the demand curve). The middle equilibrium (where cost intersects the upward sloping segment of the demand curve) is unstable because if one person...
decides to drop out of the network, then at least one of the remaining subscribers will find it unprofitable to remain connected and will leave (the value of the good is lower than the cost); but when this happens, at least another person will leave and so on until the network has no remaining members. If, on the other hand, one person decides to join, another member will find it profitable to join too, and so on until the highest equilibrium level is achieved. Therefore, in order to get the high level equilibrium from the zero equilibrium, it would not be necessary for all consumers to agree in advance to join; all is needed is to achieve the critical mass, that is, the number needed to get just beyond the unstable equilibrium.10

In this paper, we make a contribution to both the PTR and the network externalities literatures by studying how and whether a PTR is different in a market with network externalities where consumers’ expectations about demand growth is a crucial element of the market. To our knowledge, this paper is the first attempt to look at this question.

3. The Model

In principle, we would be able to study the PTR on both the upward sloping and downward sloping portions of the demand curve. However, in order to have a meaningful and fair comparison between the network case and the standard case,11 our attention will focus on the downward sloping part of the demand function. In addition, our focus will be to study the effect of a unique feature of markets with network externalities: the role of consumers’ expectation regarding the growth of the network (i.e., expectations about other consumers joining the market). In particular, it is possible that, beyond the effect caused by the curvature of demand, consumers’ expectations will affect the PTR in network markets. In the analysis that follows we take a first step in comparing the PTR in the standard monopoly and Cournot oligopoly cases to their respective network externalities counterparts by isolating the effect of consumers’ expectations from the curvature of demand effect; as we explain in more detail later, we do so by restricting the change in quantity due to a cost shock to be the same in both cases.

We first derive the demand for the network good following the well-established procedure described in Economides (1996) and Economides and Himmelberg (1995). Besides being simple, the advantage of this approach is that it nests the no-network (i.e., standard) case as a special scenario. We assume that each consumer buys only one unit of the network good and define the corresponding utility as $u(y, Q^*) = yh(Q^*)$. The term $h(Q^*)$ is a network externality function which captures the influence of network size expectations ($Q^*$) on the utility level; as is standard in the literature on network externalities, we assume that $h' > 0$ (a larger expected size of the network results in higher individual utility) and $h'' \leq 0$ (the marginal network externality is decreasing in network size). The term $y$ measures the consumer’s sensitivity to the network externality; $y$ is assumed to be uniformly distributed between 0 and 1. Importantly, the multiplicative specification of the utility function allows for different types of consumers to be affected differently by
the network externality. Given expectations \( Q' \) and price \( p \), every consumer for whom \( u(y, Q') = y h(Q') \geq P \) will purchase the good. The indifferent consumer (\( y' \)) condition is given by \( u(y, Q') = y' h(Q') = P \). Thus, all consumers with \( y > y' = P / h(Q') \) will purchase the good. Given the uniform distribution of types, aggregate demand is given by \( Q = 1 - y' \), which is equivalent to:

\[
Q = 1 - \frac{p}{h(Q')}. (1)
\]

The inverse demand function for the network good is hence \( P = (1 - Q) h(Q') \). Seen as a function of its first argument, this is just an inverse demand function, and therefore \( \partial P / \partial Q' = -h'(Q') < 0 \); however, due to the existence of network externalities, expectations also affect positively the willingness to pay: \( \partial P / \partial Q' = (1 - Q) h' > 0 \). In equilibrium, when expectations are fulfilled (\( Q = Q' \)), the demand function becomes \( P = (1 - Q) h(Q) \).

### 3.1 Monopoly

Consider a monopoly facing consumer demand \( P = (1 - Q) h(Q) \) and convex cost function \( C(Q, t) \), where \( t \) is a cost-shifting parameter. We assume that the parameter \( t \) shifts marginal cost only (and not fixed costs); this specification fits well the diverse cost pass-through scenarios studied by the literature: exchange rate changes that make inputs pricier (or cheaper), a cost synergy associated with a merger, or a per-unit tax increase. Our assumed demand functional form implies that quantity \( (Q) \) is between 0 and 1. In order to analytically manipulate the model, we need to specify a functional form for \( h(Q) \) that satisfies \( h' > 0 \), \( h'' \leq 0 \), and \( 0 < Q \leq 1 \) and that nests network and no-network demands as special cases. Specifically, we use \( h(Q) = Q^k \) with \( 0 \leq k \leq 1 \). When \( k \to 0 \Rightarrow P = 1 - Q \), which corresponds to the standard monopoly case with linear demand function; when \( k \to 1 \Rightarrow P = (1 - Q) Q \), which corresponds to the network externality case with a linear specification of the network externality function; when \( 0 < k < 1 \Rightarrow P = (1 - Q)^{Q'} \), which corresponds to the network externality case with a non-linear specification of the network externalities function.

#### Standard Monopoly Case

Given the assumed inverse demand curve, a monopolist maximizes \( \pi = P(Q) Q - C(Q, t) = (1 - Q) Q - C(Q, t) \) with first- and second-order conditions:

\[
\frac{\partial \pi}{\partial Q} = P + Q \frac{\partial P}{\partial Q} - C_v = 1 - 2Q - C_v = 0,
\]

\[
\frac{\partial^2 \pi}{\partial Q^2} = -2 - C_{vv} \leq 0.
\]
A change in the cost-shifting parameter \( t \), which affects marginal costs, will generally induce changes in the equilibrium price, output, and profit. Here we derive comparative statics results for an infinitesimal tax change. We first totally differentiate the first-order condition in (2) yielding

\[
\frac{dQ^s}{dt} = \frac{C_\omega}{-2 - C_{\omega\omega}},
\]

(3)

where the \( s \) superscript denotes the standard case. Without loss of generality, marginal costs are assumed to be a non-decreasing function of the cost shock: \( C_\omega \geq 0 \). The strict convexity condition of the cost function implies that the denominator in (3) is negative, thereby making \( \frac{dQ^s}{dt} \) also negative. The monopolist optimal absolute pass-through rate is then

\[
\frac{\partial P^s}{\partial t} = -d\frac{dQ^s}{dt} > 0.
\]

**Network Externalities Monopoly (Non-Linear Case)**

The inverse demand function \( P = (1 - Q)Q' \), which is concave, reaches its maximum when \( Q = k/k + 1 \). As stated earlier, to make a meaningful comparison between the market with a network externality and its no-externality counterpart, we focus the analysis on the downward portion of the demand curve (i.e., when \( Q > k/k + 1 \)). The monopolist’s first- and second-order conditions are:

\[
\frac{\partial \pi}{\partial Q} = P + Q \frac{\partial P}{\partial Q} - C_\omega = Q' (1 - Q)(k + 1) - Q'^{-1} - C_\omega = 0,
\]

(4)

\[
\frac{\partial^2 \pi}{\partial Q^2} = k(k + 1)Q'^{-1} - (k + 2)(k + 1)Q' - C_{\omega\omega} \leq 0.
\]

As above, we totally differentiate the first-order condition in (4) to obtain

\[
[k(k + 1)Q'^{-1} - (k + 2)(k + 1)Q' - C_{\omega\omega}]dQ - C_\omega dt = 0,
\]

which implies:

\[
\frac{dQ^{ne}}{dt} = \frac{C_\omega}{k(k + 1)Q'^{-1} - (k + 2)(k + 1)Q' - C_{\omega\omega}},
\]

(5)

where the \( NE \) superscript denotes the network externalities case. As in the standard monopoly case above, (5) is negative. The monopolist optimal absolute pass-through rate is:

\[
\frac{\partial P^{ne}}{\partial t} = \frac{\partial Q}{\partial t} \left[ kQ'^{-1} - (k + 1)Q^s \right].
\]

(6)

As in the standard monopoly case, the monopolist optimal absolute pass-through rate with network externalities is positive on the downward sloping portion of the demand curve (\( Q > k/(k + 1) \)) and negative when \( Q \) is located on the upward sloping part of the demand curve (\( Q < k/(k + 1) \)). The explanation for the latter
result (which may seem counterintuitive at first) is the following. Recall that the upward sloping part of the demand curve represents the start-up phase of the network, where the critical mass is located; it is in this portion of the demand curve that unstable equilibria arise. Figure 2 displays a network externalities demand curve and two constant marginal cost lines $C_1$ and $C_2$. Consider the unstable equilibrium, $Q^A$, located at the left intersection of $C_1$ and the demand curve. An upward shift in the marginal cost function to $C_2$ (a cost shock) pushes the network toward the zero stable equilibrium (see the arrows in Figure 2) since $Q^A$ is now below the new critical mass (given by the intersection of the marginal cost $C_2$ and the demand function) and the network naturally goes to the zero-quantity equilibrium.

An important feature of our paper is the approach used to compare the PTR in the standard and network externality cases. The prior literature on PTR has shown that the magnitude of the PTR is, by and large, due to the curvature of demand. Since we are interested in isolating the effect of the network externality on the PTR, we need to ensure that the difference in the curvature of demand between the two cases of interest is not responsible for our finding. To do this, we study the comparative static of interest, $dp/dt$, under the following condition:

$$\frac{dQ^t}{dt} = \frac{dQ^\text{NE}}{dt}. \tag{7}$$

That is, we fix the variation in quantity due to an infinitesimal cost shock ($t$) change to be the same in the two cases of interest. This procedure allows us to isolate the effect of network externalities on the PTR while keeping the curvature of demand constant for the two cases of interest. In other words, if we do not impose this condition, a cost shock will not only affect the equilibrium price by different amounts in the two cases but also affect quantities differently due to differences in the demand functional form.
**Proposition 1.** For the same change in quantity (given by an infinitesimal variation in the cost shock parameter), the pass-through rate in the presence of network externalities is lower than the pass-through rate in the standard monopoly case.

**Proof.** When (7) holds:

\[
\frac{dP^s}{dt} \geq \frac{dP^{ne}}{dt} \quad \text{if} \quad \frac{\partial Q}{\partial t} \geq \frac{\partial Q}{\partial t} \left[ kQ^{1-1} - (k+1)Q^t \right],
\]

which occurs when \(1 + kQ^{1-1} - (k+1)Q^t > 0\). After mathematical manipulation, the condition needed for (8) to hold is \(kQ^t(1 - Q) + Q(1 - Q^t) > 0\), which is always true.

The intuition for this result is as follows. A positive cost shock contracts the supply curve, which, in turn, implies a smaller equilibrium quantity and a higher price (on the downward sloping portion of the demand curve). We call this the “standard effect”. With network externalities, there is an additional effect that runs opposite to the standard effect: if the quantity decreases (as a result of the positive cost shock), the size of the network decreases, and the willingness to pay by the marginal consumer also decreases, thereby inducing a decrease in equilibrium price. This is because the utility of having a network good positively depends on the number of other consumers connected to the network. This “network externalities” effect reduces the cost pass-through rate magnitude that would be observed otherwise.

**Network Externalities: Linear Case**

The inverse demand function faced by a monopolist producing the network good in this case is \(P = (1 - Q)Q\). This function reaches its maximum at \(Q = 1/2\) and has first- and second-order conditions:

\[
\frac{\partial \pi}{\partial Q} = 2Q - 3Q^2 - C_\varphi = 0 \quad \text{and} \quad \frac{\partial^2 \pi}{\partial Q^2} = 2 - 6Q - C_{\varphi,\varphi} \leq 0.
\]

Totally differentiating the first-order equilibrium condition obtains \([2 - 6Q - C_{\varphi,\varphi}]dQ - C_\varphi dt = 0\), which implies \(dQ^{LNE}/dt = C_\varphi / 2 - 6Q - C_{\varphi,\varphi}\), where the \(LNE\) superscript denotes the linear network externalities case. This last expression is negative. Therefore, the corresponding monopolist optimal absolute pass-through rate is:

\[
\frac{\partial P^{LNE}}{\partial t} = \frac{\partial Q}{\partial t} [1 - 2Q],
\]

which is positive for \(Q > 1/2\) (i.e., on the downward sloping part of the demand curve). For the same variation in quantity \(dQ^s/dt = dQ^{LNE}/dt\) as in the non-linear case, it is easy to check that the pass-through rate is greater than the pass-through rate in the network externality case. Specifically, \(dP^s/dt = dP^{LNE}/dt\) if
\[-dQ^*/dt - (dQ^{*,e}/dt)(1 - 2Q) > 0; \text{ that is, if } 1 + 1 - 2Q > 0, \text{ which is always true.}\]

3.2 Cournot Oligopoly with \( n \) Firms

Here, we consider \( n \) identical firms producing a homogeneous good, with the same cost structure as in the monopoly case.

**Standard Cournot Oligopoly Case**

The demand function in this case is given by \( P = 1 - Q = 1 - (q_i + q_n) \). Firm \( i \) sets its level of output to maximize its profit as \( \pi_i = P(Q)q_i - C(q_i) \). The (symmetric) first-order condition for profit maximization results in

\[
\frac{dq_i}{dt} = C_q - n - 1 - C_{q_t} < 0, \quad \text{with corresponding pass-through rate given by} \quad \frac{\partial P}{\partial t} = -ndq/dt > 0.
\]

**Network Externalities Cournot Oligopoly: Non-Linear Case**

The inverse demand function in this case is \( Q^*P = Q_Q^*Q^* - kQ^*Q^* - q_i^*Q^* - q_j^*Q^* \). Firm \( i \)'s first- and second-order conditions for profit maximization are:

\[
\frac{\partial \pi_i}{\partial q_i} = P + q_i [kQ^*i - (k + 1)Q^*] - C_{q_i} = Q^* - Q^*i + kq_iQ^*i - (k + 1)q_iQ^* - C_{q_i} = 0,
\]

\[
\frac{\partial^2 \pi_i}{\partial q_i^2} = 2kQ^*i - 2(k + 1)Q^* - k(k - 1)q_iQ^* - k(k + 1)q_iQ^* - C_{q_{ii}} \leq 0.
\]

Thus, in (the symmetric) equilibrium (i.e., \( q_i = q \) for all firms) the first-order condition becomes \( (nq)^* - (nq)^*i + kq(nq)^*i - (k + 1)q(nq)^* - C_{q_i} = 0 \). To analyze an infinitesimal tax change, we need to totally differentiate the last first-order equilibrium condition, which yields

\[
[knQ^*i - (k + 1)nQ^* + kQ^* - (k + 1)Q^* - C_{q_{ii}}]dq - C_{q_i}dt = 0 \quad \text{or:}
\]

\[
dq^{*,e}/dt = \frac{C_{q_i}}{knQ^*i - (k + 1)nQ^* + kQ^* - (k + 1)Q^* - C_{q_{ii}}}. \tag{9}
\]

Recall that marginal costs are assumed to be a non-decreasing function of the cost shock parameter (\( C_{q_{ii}} \geq 0 \)). Conversely, local stability of the equilibrium implies that the denominator in (9) is negative, thereby making \( dq^{*,e}/dt \) also negative.

Each firm’s optimal absolute pass-through rate is \( \partial P^{*,e}/\partial t = n(\partial q_i/\partial t)[kQ^*i - (k + 1)Q^*] \), which is positive if \( kQ^*i - (k + 1)Q^* < 0 \); that is, if \( Q > k/k + 1 \) (i.e., on the downward sloping part of the demand function), as in the monopoly case.

Consider the same variation in quantity in the standard and network (oligopoly) cases due to an infinitesimal variation in the tax parameter (i.e., \( dq^*/dt = dq^{*,e}/dt \)).
Proposition 2. When firms compete in quantities (Cournot competition) and \( dq^t / dt = dq^{ne} / dt \), the optimal pass-through rate with network externalities is lower than the optimal pass-through rate in the standard case.

Proof. It is easy now to compare the pass-through rate with and without network externalities when \( dq^t / dt = dq^{ne} / dt \) holds:

\[
\frac{dP^s}{dt} \geq \frac{dP^{ne}}{dt} \text{ when } -\frac{\partial dQ}{\partial t} \geq \frac{\partial dQ}{\partial t} \left[ kQ^{t-1} - (k + 1)Q^t \right],
\]

which implies that \( 1 + kQ^{t-1} - (k + 1)Q^t > 0 \); it is straightforward to verify that this condition always holds.

Results are similar in Cournot with a linear network externalities specification. Appendix A1 contains these results.

3.3 Robustness: Non-Pure Network Good

To investigate how robust are our results to a variation in the definition of a network good, we consider a network externalities (linear) function for a non-pure network good:

\[
h(Q) = s + Q,
\]

where \( s \) represents the non-network benefits that a good provides: it measures the willingness to pay for a unit of the good when there are no other units sold. When \( s = 0 \), we are back to a market to a pure network good (discussed above).

Monopoly Case

The inverse demand function faced by a monopolist producing the network good is \( P = (1 - Q)(s + Q) \). This function reaches its maximum at \( Q = 1 - s/2 \). The monopolist’s profit is \( \pi = P(Q)Q - C(Q) \), with first- and second-order conditions:

\[
\frac{\delta \pi}{\delta Q} = P + Q \frac{\delta P}{\delta Q} - C_o = s - 2sQ + 2Q - 3Q^2 - C_o = 0,
\]

\[
\frac{\delta^2 \pi}{\delta Q^2} = 2 - 2s - 6Q - C_{oo} \leq 0.
\]

Totally differentiating the first-order condition obtains \( [2 - 2s - 6Q - C_{oo}] dt = 0 \). After rearranging terms, this last expression becomes \( \frac{dQ^{ne}}{dt} = C_o^2 / 2 - 2s - 6Q - C_{oo} < 0 \). Accordingly, the monopolist’s optimal absolute pass-through rate is \( \frac{\delta P^{ne} / \delta t}{\delta t} = (C/Q) \delta Q / \delta t [1 - s - 2Q] \), which is positive for \( Q > 1 - s/2 \) (a quantity level located on the downward sloping part of the demand curve). In the standard monopoly case, the inverse demand function is \( P = (1 - Q)s \), which results in:
\[
\frac{dQ}{dt} = \frac{C_0}{2s-C_{\infty}} \quad \text{and} \quad \frac{\partial P}{\partial t} = -s \frac{\partial Q}{\partial t}.
\]

**Proposition 3.** When \( \frac{dQ}{dt} = \frac{dQ^\infty}{dt} \), the pass-through rate in the standard monopoly case with a non-pure network good is greater than the pass-through rate in the corresponding network externality case.

**Proof.** The condition \( \frac{dQ}{dt} = \frac{dQ^\infty}{dt} \) holds if \( Q^* = 1/3 \). Therefore, \( \frac{dP}{dt} = \frac{dP^\infty}{dt} \) if \( -s(dQ/dt) - (dQ^\infty /dt)(1-s-2Q) > 0 \), that is, if \( Q < 1/2 \), which is always true because \( Q^* = 1/3 \).

Further, it is easy also to check that the same proposition holds in the Cournot oligopoly case with a non-pure network good. Appendix A2 contains the proof.

4. Welfare Analysis

The motivation for carrying out welfare comparative statics is justified by the importance of per-unit taxes in the pass-through literature. We consider two welfare measures: (1) the excess burden of a per-unit tax with and without network externalities and (2) welfare variations (due to a per-unit tax change) in a standard monopoly market and in a monopoly with network externalities. The difference between the excess burden and the welfare analyses is that the former takes into account tax revenues (thereby being a comparison of total societal welfare—consumers, producers, and the government), while the latter only focuses on the welfare of consumers and producers (i.e., it excludes tax revenues). We carry out this second calculation since in many cases policymakers may be interested in the effects of per-unit taxes on each group of agents rather than in the sum.

4.1 Excess Burden

The excess burden of taxation is the efficiency cost, or deadweight loss, associated with taxation. In this section we compare, in a monopoly situation, the incremental excess burden of a per-unit tax with and without network externalities. Figure 3 illustrates a standard inverse demand curve and the corresponding marginal revenue (MR) curve for a monopolist. Suppose marginal costs (MC) are constant. The initial marginal cost is \( C_0 \), which intersects MR where \( Q = Q_0 \); the corresponding price is \( P_0 \). Prior to a per-unit tax increase, the excess burden is the area below (greater than zero because price exceeds MC).

After the imposition of a tax (which shifts MC up to \( C_{\text{tax}} \)), price increases to \( P_1 \) and quantity decreases to \( Q_1 \), while the incremental excess burden is the area \( abcd \); algebraically, this area can be approximated by the following formula:
To compare the monopoly situation with and without network externalities, we consider, as in the analysis above, the same variation in the quantity due to the imposition of a tax, that is \( \frac{dQ}{dt} = \frac{dQ}{dt}^{\text{NES}} \).

**Proposition 4.** The incremental excess burden of an excise tax is greater in the standard monopoly case than in the corresponding network externalities case.

**Proof.** The incremental excess burden due to a tax positively depends on \( \frac{\partial P}{\partial t} \). Given that we have shown that \( \frac{dP}{dt} > \frac{dP}{dt}^{\text{NES}} \), this result implies that the deadweight loss associated to a tax is always lower in a market with network externalities.

This result is important from a public policy perspective since it states that if governments need to decide between taxing a market that is subject to important network externalities and a standard market (and if policymakers care about deadweight loss), taxation of the network market should be avoided. We further discuss the implication of this result in the conclusion.

### 4.2 Welfare Variation

We now consider the variation in welfare in a monopoly market in both the standard and network cases when there is a change in the per-unit tax. The idea is to see how network externalities affect the variation in welfare for both consumers and producers.
Standard Monopoly Case

Since in the standard monopoly case the demand function is linear, \( CS \) is given by the expression
\[
2 \left( \frac{1}{2} \right) \left( 1 - P \right) = (1/2)Q^2,
\]
and its change due to a per-unit tax variation is \( \partial CS / \partial t = Q(\partial Q / \partial t) < 0 \).

The monopolist profit is \( \pi^t = Q - Q^2 - C(Q, t) \) and its variation to a tax change is \( \partial \pi^t / \partial t = (\partial Q / \partial t)(1 - 2Q - C_v) - (\partial C / \partial t) < 0 \). The sign of this last expression depends on the magnitude of \( C_v \) and \( \partial C / \partial t \), therefore it is a priori not predictable. The variation in the welfare due to a variation in a per-unit tax, after some manipulation, is then \( \partial W^t / \partial t = (\partial Q / \partial t)(1 - Q - C_v) - (\partial C / \partial t) \). While the expression in parenthesis in the last equation can mathematically be negative, thereby making \( \partial W^t / \partial t \) (possibly) positive, it is economically unlikely. Note that the term \( 1 - Q - C_v \) is simply the mark-up since \( 1 - Q = P \), and we expect a monopolist to be operating with a positive mark-up.

Network Externalities: Non-Linear Monopoly Case

Let us start with the case where the network externalities function is not linear, that is \( h(Q) = Q^k \rightarrow P = (1 - Q)Q^k \). Welfare \( (W) \) is the sum of the consumer surplus \((CS)\) and the monopolist’s profit \((\pi)\): Consequently, the variation in the welfare due to an infinitesimal variation in a per-unit tax is:

\[
\frac{\partial W}{\partial t} = \frac{\partial CS}{\partial t} + \frac{\partial \pi}{\partial t}.
\]

In this non-linear case, the \( CS \) is given by the following expression:

\[
CS^ne = \int PdQ - PQ = \frac{1}{k + 1}Q^{k+1} - \frac{1}{k + 2}Q^{k+2} - Q^{k+1}Q^{k+2}.
\]

Thus, the variation in \( CS \) due to an infinitesimal variation in the per-unit tax is:

\[
\frac{\partial CS^ne}{\partial t} = \frac{\partial Q}{\partial t} \left[ (k + 1)Q^{k+1} - kQ^2 \right],
\]

which is always negative on the downward sloping part of the demand curve.

The monopolist profit is \( \pi^ne = Q^{k+1} - Q^{k+2} - C(Q, t) \) and its change due to a variation in the per-unit tax is \( \partial \pi^ne / \partial t = (\partial Q / \partial t)((k + 1)Q^k - (k + 2)Q^{k+1} - C_v) - (\partial C / \partial t) \). The sign of this expression is not predictable for the same reason stated earlier. The resulting change in welfare due to a variation in a per-unit tax, after some manipulation, is \( \partial W^ne / \partial t = (\partial Q / \partial t)((Q^k - Q^{k+1} - C_v) - (\partial C / \partial t) \). By the same reasoning we used in the standard monopoly case above, this expression will be negative.

**Proposition 5.** Under the same change in quantity due to an infinitesimal variation in
the cost shock parameter (i.e., \( dq^* / dt = dq^{\text{ne}} / dt \)), the change in total welfare in the network externalities monopoly case is greater than the change in welfare in the standard monopoly case.

**Proof.** The comparison of the welfare variation is given by:

\[
\frac{\partial W^{\text{xe}}}{\partial t} > \frac{\partial W^{*}}{\partial t} \Leftrightarrow \frac{\partial Q}{\partial t} \left( Q^* - Q^{*\text{ne}} - C_o \right) - \frac{\partial C}{\partial t} > \frac{\partial Q}{\partial t} Q + \frac{\partial Q}{\partial t} \left( 1 - 2Q - C_o \right) - \frac{\partial C}{\partial t},
\]

which, after manipulation, implies \((1 - Q)(Q^* - 1) < 0\), which is always true at \(dQ^* / dt = dQ^{\text{xe}} / dt\).

We also show that, when comparing the variation in the CS in the standard and network externality (monopoly) cases, the condition \( \partial CS^{\text{xe}} / \partial t > \partial CS^{*} / \partial t \) always holds, while the relation between the variation in the two monopoly profits is indeterminate; importantly, (10) also holds in the linear externality case with a pure and non-pure network good, as well as in the Cournot oligopoly case.

**Implications**

Equation (10) indicates that while welfare losses in the case of a per-unit tax increase are larger (more negative) in the standard case (consistent the excess burden analysis), welfare gains (due to an increase in subsidies) are also larger in the standard case (inconsistent with the excess burden analysis). The reason for the discrepancy with the excess burden analysis (in the case of a per-unit subsidy increase) is that the deadweight loss analysis includes both changes in tax revenues (or subsidy payments) and changes in welfare. To see how to reconcile these analyses, recall that a price decrease, as a result of a per-unit subsidy increase, is larger in the standard case; this means that subsidy payments are larger in the standard case (large enough to offset the larger welfare gain in the standard case).

Since the sign of profit change as a result of a per-unit tax change is indeterminate, the result in (10) is mainly driven by the smaller (in absolute value) consumer welfare change; since we have “controlled for” the role of demand curvature on the PTR, this result means that a smaller PTR in the network case is due to consumers’ expectations about network growth.

5. Conclusions

Cost pass-through is a key economic comparative static. The results of this paper contribute to the growing and influential cost pass-through literature by showing that network externalities can be an important determinant of the pass-through rate. Cost pass-through can be influenced by both demand and supply conditions; since network externalities are fundamentally a demand-side
phenomenon, our analysis can be classified as a demand-driven factor of cost pass-through. Specifically, the cost pass-through rate is smaller in a market with network externalities than in a no-network-externality counterpart; an important feature of our study is that this result is robust to several demand specifications and market structures and that it does not depend on the difference in curvature of demand between the two cases considered.

Our results also suggest that when facing a per-unit tax decrease (i.e., a subsidy increase), consumers would, all else equal, prefer to consume a standard good (over a network good); but they would prefer a network good (over a standard good) otherwise. The reason for this finding is that, given the same quantity change, a per-unit subsidy increase (decrease) will decrease (increase) price by a smaller amount in the network good case than in the standard case.

Finally, our analysis can be informative for policymaking. If intervention via per-unit subsidies is chosen, and if the government only cares about welfare (and not tax revenue, or subsidy payments), it would prefer, all else equal, to intervene in a standard market rather than in a network good market (if it had to face such a choice). Conversely, if per-unit taxes are chosen, and if welfare is the only consideration, the government would prefer a network market over a standard market. On the other hand, if government cares about deadweight loss, it will prefer a network good market over a standard good market regardless of the type of intervention (per-unit tax or per-unit subsidy). Finally, government intervention through taxation can be critical if the network is facing the upward sloping section of the demand curve: an increase of the excise tax can make the network disappear, but a subsidy can propel it to stable growth.

Appendix


In this case, the inverse demand function is \( P = (1 - Q)Q \) with \( Q = q_i + q_j \) and \( q_{ij} = \sum_{j \neq i} q_j \). Firms have the same cost structure as in the monopoly case. Firm \( i \) has first- and second-order conditions:

\[
\frac{\partial \pi_i}{\partial q_i} = P + q_i[1 - 2Q] - Cq_i = Q - Q^2 + q_i - 2q_iQ - Cq_i = 0,
\]

\[
\frac{\delta^2 \pi_i}{\delta q_i^2} = 2 - 4Q - 2q_i - C \leq 0.
\]
Thus, in the symmetric equilibrium, the firm’s necessary condition becomes 
\[ nq - (nq)^2 + q - 2n^2q - C_q = 0 \]. Totally differentiating the first-order equilibrium condition, we have \[ [n + 1 - 2n^2 - 4nq - C_n]dq - C_q dt = 0 \], which, after rearranging terms, becomes:

\[
\frac{dq^N}{dt} = \frac{C_w}{n + 1 - 2n^2 - 4nq - C_q},
\]

where \( C_w \geq 0 \); local stability of the equilibrium implies that the denominator of (a) is negative. Thus \( dq^N / dt \) is negative. The firm optimal absolute pass-through rate is thus \( \partial P^N / \partial t = n(\partial q^N / \partial t)(1 - 2nq) \), which is positive if \( 1 - 2nq < 0 \), that is, if \( Q > 1/2 \) (i.e., on the downward sloping part of the demand function). As in the monopoly case, we consider the same variation in quantity (in both the network and standard cases) due to an infinitesimal variation in the cost-shock parameter. Then, the equation \( dq^N / dt = dq^N / dt \) is satisfied for \( Q^* = n + 1/n + 2 \). It is easy to show that \( dP^N / dt \geq dP^S / dt \).

**A2. Cost pass-through analysis in Cournot oligopoly with non-pure network goods.**

The inverse demand function is \( P = (1 - Q)(s + Q) \) with \( Q = q_1 + q_2 \), and \( q_2 = \sum_{(i\neq j)} q_j \). Firms have the same cost structure as in the monopoly case. Totally differentiating the first-order condition, we obtain:

\[
\frac{dq^N}{dt} = \frac{C_w}{(1 - s)(1 + n) - 2nq(n + 2) - C_w}.
\]

The firm’s optimal absolute pass-through rate is thus \( \partial P^N / \partial t = n(\partial q^N / \partial t)(1 - s - 2nq) \), which is positive if \( 1 - s - 2nq < 0 \), that is, if \( Q > 1 - s/2 \) (i.e., on the downward sloping part of the demand function).

In the standard Cournot case, the demand function is \( P = (1 - Q)s \) with \( Q = q_1 + q_2 \), and \( dq^N / dt = C_w (1 - s(1 + n) - C_w \), and the optimal PTR is \( \partial P^N / \partial t = -sn(\partial q^N / \partial t) \).

As stated throughout the paper, we consider the same variation in quantity (in both the network and standard cases) due to an infinitesimal variation in the cost-shock parameter. Then, the equation \( dq^N / dt = dq^N / dt \) is satisfied for \( Q^* = n + 1/2(n + 2) \). It is easy to check that \( dP^N / dt \geq dP^S / dt \). Indeed \( dP^N / dt \geq dP^S / dt \) if \( -sn(\partial q^N / \partial t) > n(\partial q^N / \partial t)(1 - s - 2nq) \), which implies that \( Q < 1/2 \) is always true because \( Q^* = n + 1/2(n + 2) \).

**A.3 Welfare analysis, monopoly case with linear network externality specification.**

Here, we derive the variation in consumer surplus (\( CS \)), profit, and welfare in the monopoly case with a pure network good as a result of a per-unit tax change. (The derivation is similar with a non-pure network good.) The demand function is \( P = (1 - Q)Q \), while consumer surplus is given by \( CS^N = \int PdQ - PQ \)
\[
= (2/3)Q^3 - (1/2)Q^2. \text{ The variation in the } CS \text{ due to an infinitesimal variation in the tax parameter is } \frac{\partial CS}{\partial t} = Q(\partial Q/Qt)(2Q - 1), \text{ which is negative for } Q > 1/2; \text{ that is, on the downward sloping part of the inverse demand curve.}
\]

The monopolist profit is \( \pi^M = Q^3 - C(Q, t) \) and its change due to a variation in the tax is
\[
\frac{\partial \pi^M}{\partial t} = (\partial Q/Qt)((2Q - 3Q^2 - C_0) - (\partial C/\partial t) \text{ with a sign that is not a priori predictable because it depends on the magnitude of } C_0 \text{ and } \partial C/\partial t.
\]

Recalling the expression for \( CS \) and profit in the standard case, we can compare the variation in the \( CS \), profit, and welfare due to an infinitesimal variation in the tax parameter in both cases. When \( \partial Q^M/Qt = \partial Q^M/\partial t \) (which is true when \( Q^* = 1/2 \)):

\[
\frac{\partial CS^M}{\partial t} > \frac{\partial CS^S}{\partial t} \text{ because } Q^M(\partial Q/Qt)(2Q - 1) > Q^S(\partial Q/Qt); \\
\frac{\partial \pi^S}{\partial t} < \frac{\partial \pi^S}{\partial t} \text{ because } \frac{\partial Q}{\partial t}(2Q - 3Q^2 - C_0) - \frac{\partial C}{\partial t} < \frac{\partial Q}{\partial t}(1 - 2Q - C_0) - \frac{\partial C}{\partial t} \text{ and } \\
\frac{\partial W^S}{\partial t} > \frac{\partial W^S}{\partial t}.
\]


The inverse demand function in this case is \( P = (1 - Q)Q^i \) with \( Q = q_i + q_{-i} \) and \( q_{-i} = \sum_{j \neq i} q_j \). Consider the symmetric case \( q = q_i = q_j \). The \( CS \) is:

\[
CS^S = \int Pdq - Pq = \int (nq)^i dq - \int (nq)^i dq - q(nq)^i + q(nq)^{i-1}
\]

\[
= \frac{1}{k + 1} n^i q^{i-1} - \frac{1}{k + 2} n^{i+1} q^{i+2} - n^i q^{i+1} + n^{i+2} q^{i+2}.
\]

The variation in the \( CS \) due to an infinitesimal variation in the tax parameter is
\[
\frac{\partial CS^S}{\partial t} = (\partial q/Qt)((nq)^i - (nq)^{i-1} - (k + 1)(nq)^i + (k + 2)(nq)^{i+1}), \text{ which is always negative on the downward sloping part of the inverse demand curve.}
\]

The profit of a given firm is \( \pi^S = q(nq)^i - q(nq)^{i-1} - C(Q, t) \) and its change due to a variation in the tax is
\[
\frac{\partial \pi^S}{\partial t} = (\partial q/Qt)((k + 1)(nq)^i - (k + 2)(nq)^{i+1} - C_0) - \frac{\partial C}{\partial t}.
\]

Let us now consider the standard oligopoly case. The consumer surplus is \( CS^S = (1/2)nq^2 \) and the corresponding profit is \( \pi^S = q(1 - nq) - C(Q, t) \), with
\[
\frac{\partial CS^S}{\partial t} = q(\partial q/\partial t) \quad \text{and} \quad \frac{\partial \pi^S}{\partial t} = (\partial q/\partial t)(1 - 2nq - C_0) - \frac{\partial C}{\partial t}.
\]

After imposing \( \frac{\partial Q^S/\partial t = \partial Q^S/\partial t} \), we can say that \( \frac{\partial W^S/\partial t > \partial W^S/\partial t} \) holds if:
\[
\frac{\partial q}{\partial t} \left[ (nq)^k - (nq)^{k+1} - (k+1)(nq)^k + (k+2)(nq)^{k+1} \right] \\
+ \frac{\partial q}{\partial t} \left[ (k+1)(nq)^k - (k+2)(nq)^{k+1} - C_o \right] - \frac{\partial C}{\partial t}, \\
> nq \frac{\partial q}{\partial t} + \frac{\partial \pi^2}{\partial t} = \frac{\partial q}{\partial t} (1 - 2nq - C_o) - \frac{\partial C}{\partial t},
\]
which is true if \((Q^* - 1)(1 - Q) < 0\), a relationship that always holds. It is straightforward to prove the same result with the linear specification of the network externalities function.

Notes

1. Cost shocks are common in industries characterized by network effects. An example of this is the telecommunications industry, which is often subject to exogenous shocks due to technology innovations (a negative cost shock) or to volatile exchange rates that affect the price of imported inputs (such as equipment).

2. We provide references to more comprehensive reviews throughout this section.

3. Since per-unit tax changes and cost shocks have equivalent mathematical treatments, public economics studies (on taxes) and industrial organization studies (on costs) have significant overlap; for this reason, although we only refer to cost pass-through, we interchangeably refer to studies in both literatures in this section.

4. An alternative concept used to study this issue is pass-through rate elasticity (PTE), which is defined as \((\partial p / \partial c)(c/p)\). The concept PTR is commonly used by the industrial organization literature, whereas PTE is typically adopted by the international economics literature. Without loss of generality for our analysis below, we adopt the former.

5. As noted by Anderson et al. (2001), Seade (1997), and Fullerton and Metcalf (2002), however, in these cases the conditions for over- or under-shifting also depend on the demand curvature.

6. For a recent review of factors affecting PTR see Kosicki and Cahill (2006).

7. While under-shifting in this literature focuses on PTE, PTR estimates in this literature seem to suggest that under-shifting is still a common finding (e.g., Nakamura and Zerom, 2009; Goldberg and Hellerstein, 2008).

8. While there have not been attempts to reconcile these two apparently opposing views, there are two likely reasons. First, the two literatures consider different types of cost shocks: international economists study exchange rates whereas public economists are primarily concerned with excise taxes. Second, the industrial organization literature focuses on PTR (i.e., \(\partial p / \partial c\)), while international economists have (typically) studied PTR (i.e., \((\partial p / \partial c)(c/p)\)). Clearly, unless mark-ups are zero, PTR will be smaller than PTE.

9. Delipalla and O’Donnell (2001) also analyze the PTR in the cigarette industry.

10. Rogers (2003) defines the critical mass as the minimal number of subscribers (adopters) of an interactive innovation for the further rate of adoption to be self-sustaining; that is, network effects can generate multiple stable equilibria separated by an unstable one, the critical mass.

11. To simplify exposition we will henceforth often refer to the network externality case simply as the “network” case.
12. This assumption diverges from that of Katz and Shapiro (1985) and Cabral (1990), who use an additive utility specification, so that all consumers receive the same benefit from the network externality.

13. \( C^* = \frac{\partial C}{\partial Q} > 0 \) and \( C''^* = \frac{\partial^2 C}{\partial Q^2} > 0 \).

14. The last two cases represent a “pure network good” (i.e., the good has no stand alone value). We later address the non-pure network good case.

15. To see this, note that \( P^r > 0 \) when \( 0 \leq Q < k(k+1) \) while \( P^l < 0 \) when \( k(k+1) < Q \leq 1 \).

16. The second order condition \( \frac{\partial^2 \pi}{\partial Q^2} = k(k+1)Q^2 - (k+2)(k+1)Q - C_{Q \pi} \leq 0 \) holds if \( Q > k(k+2) \). Therefore it always holds when \( Q > k(k+1) \) (since \( k(k+2) \) is smaller than \( k(k+1) \)), where we concentrate our analysis below.

17. Specifically, it is straightforward to check that the denominator of the (5) is always negative for the case that is of interest for our analysis: \( k(k+1) < Q \leq 1 \).

18. \( Q^r = 1/3 \) is on the downward sloping part of the demand function if \( 3k-1 > 0 \), or when \( k > 1/3 \) (a condition that is not too restrictive).

19. An example of this is when policymakers give more weight to consumer welfare, as is the case in antitrust policy.

20. To be consistent with our earlier analysis, we compare the two cases of interest on the downward sloping portion of the demand curve (i.e., where the two cases are comparable).

21. In comparing the two cases \( CS \), after algebraic manipulation, it can be shown that \( kQ^r(Q-1) + Q(Q^r - 1) < 0 \) always holds.

22. See appendices A3 and A4.

23. Recall that \( \frac{\partial W}{\partial t} \) is negative.

24. We do recognize, however, that policymaking is subject to subtleties that would make the following implications more or less relevant. For example, the policymaker may have objectives other than welfare or deadweight loss (tax revenue, or consumer welfare, for example). Also, it is obvious that policymakers do not always face taxation decisions among the two types of markets we consider.

25. \( Q^r \) lies on the downward sloping portion of the demand curve if \( s(\pi + 2) - 1 > 0 \).

References


