Evaluate the Effectiveness of Manager Compensation

Cheng-Feng Cheng

Department of International Business, Asia University, Taiwan

Abstract

This study develops a conceptual model for compensation from firm and manager perspectives. We identify the negotiating range for a compensation plan and adopt a dynamic bargaining game to optimize the equilibrium of base salary and bonus.

Key words: game theory; optimization; compensation; negotiation; risk analysis

JEL classification: C73; J33; M12

1. Introduction

Recently, the internationalization of business activities has drawn a lot of attention to the incentive mechanism of managers. Influential studies of manager compensation can be traced to 1948, when Basset hired Arch Patton of the McKinsey Management Consultant Firm to study the issue of underpayment of executives (Milkovich and Newman, 1999). Barkema and Gomez-Mejia (1998) later argued that research on manager compensation has accumulated over 300 studies. It is suggested that compensation is a critical part of the strategic thrust of human resource management in practice (Andergassen, 2008; Bengtsson and Hand, 2011; Crutchley and Minnick, 2012). These tremendous amounts of compensation studies can be generally divided into two major streams. One addressed compensation contracts for executives or managers (e.g., Barkema and Gomez-Mejia, 1998; Bengtsson and Hand, 2011; Carpenter and Sanders, 2002; Cyert et al., 2002), whereas the other explored the compensation for salesforces or laborers (e.g., Basu et al., 1985; Bronchetti, 2012; Crutchley and Minnick, 2012; Giummo, 2010). The first stream mainly focused on evaluating the determinants and consequences of the executive compensation. Few studies have supported their arguments with theoretical modeling. Cyert et al. (2002) is one of critical articles that incorporate both theoretical and empirical content to optimize compensation schemes.

However, several research gaps remain for further validation. First, although numerous studies have engaged in compensation-related studies and identified significant determinants of executive compensation, most of these studies tended to focus on either the perspective of the firm or that of the managers. Very few studies
have taken both perspectives into consideration. A more comprehensive theoretical model that encompasses details of reality and creates more concise representations of reality for description, prediction, or optimization is yet to be determined (Shugan, 2002). This article employs a mathematical model to add more strategic value to compensation studies.

Second, one line of theoretical inquiry that has been offered is the BLSS model developed by Basu et al. (1985). The model studies how five predictor variables (uncertainty, marginal cost of production, expected guaranteed utility, effectiveness of effort, and base sales level) affect the optimal contract, firm profits, effort, and expected income. The assumption of the BLSS model with respect to the utility functions of income for the salesforce conforms to diminishing marginal utility for incomes and the utility of risk-averse individuals. Joseph and Thevaranjan (1998) extended the BLSS model and restricted the utility functions for the salesperson to be negative exponential. Although the negative exponential utility function is consistent with the assumption of the BLSS model (i.e., the law of diminishing marginal utility and the utility of risk aversion), the function may violate the convention that income is good goods for individuals. It is noted that individuals are risk averse over the positive range of outcomes. In other words, individuals are unwilling to work if the utility of income is negative. This study proposed an alternative model using the natural logarithmic function to replace the negative exponential utility function. It is believed that the natural logarithmic utility function is more reasonable since no negative will result. The natural logarithmic utility function also meets the assumption of the BLSS model.

Third, while the BLSS model using uncertainty, marginal cost of production, expected guaranteed utility, effectiveness of effort, and base sales level as the predictor variables to affect sales force compensation, other variables including retainable income, bonus variance, risk aversion, expected value of performance-based determinants, total performance, and quota-base are also important factors influencing managers compensation.

The purpose of this article is to integrate critical determinants of executive or manager compensation and establish a distinctive model to derive the optimal compensation from the perspectives of firms and managers. Despite conflicting evidence in empirical studies, they do not distort our optimal solution in that we do not restrict the patterns of these determinants in our assumptions. This article is distinctive in a number of ways. First, the study integrates the determinants of compensation in attempting to specify the typology of the determinants rather than to clarify and explain the mixed results of previous studies. Second, the study uses the nature logarithmic function, which distinguishes it from previous compensation models for salesforces. The paramount advantage of this function is that it is consistent with economic assumptions, including the positive utility of good goods, the law of diminishing marginal utility, and utility of risk aversion. Third, compensation schemes of managers should be quite different from those of salesforces; this study integrates managerial discretion, human capital, social capital, charisma, tenure, stock holding, and firm size, performance, diversification, and risk
as the performance evaluation variables of managers. Finally, the optimal solution involves the both firm and manager perspectives. Agency theory indicates that agents always desire to maximize their wealth, but this goal may contradict the principal’s interest. Additionally, the firm needs to balance the benefits of keeping the talents and encouraging their contributions, as well as the costs of remuneration to design optimal compensation schemes. Thus, the optimal compensation schemes for both sides claim our attention.

The remainder of this article is divided into several parts. First, we review the literature to discuss not only the compensation schemes appropriate for the manager, but also the determinants of compensation. Next, we describe the assumptions and develop the model. Finally, we draw several conclusions from the optimal solution.

2. Compensation Determinants and Schemes

Before we discuss the determinants of executive compensation, it’s necessary to distinguish research on manager compensation from that of Chief Executive Officer (CEO). Although many studies of executive compensation use the terms executive and manager interchangeably, most confined their analysis to CEOs (e.g., Carpenter and Sanders, 2002; Bengtsson and Hand, 2011; Bizjak et al., 2011; Liu and Mauer, 2011). Bengtsson and Hand (2011) focused on fast growing young US firms and proposed that CEO cash compensation would be higher for private venture-backed firms that have raised a higher quantity and quality of venture capital financing. Bizjak et al. (2011) suggested that when firms deviate from the economic model of peer firm choice, they tend to pick larger firms with higher CEO pay. Carpenter and Sanders (2002) pointed out that the determinants and consequences of manager compensation plans may be isomorphic with those of the CEO. Furthermore, they provided evidence that CEO pay level and structure will be strongly but imperfectly related to those same aspects of manager pay. Accordingly, we loosely amass the determinants of compensation from previous studies.

Our model of manager payment involves two major sets of determinants of compensation: the market factors and the political factors (Finkelstein and Hambrick, 1989). The market factors describe the contributions of managers to the performance of the firm. We categorize the market factors further according to the criterions consisting of firm-specific factors, such as firm size, performance, diversification, and risk (e.g., Bizjak et al., 2011; Conyon et al., 2001; Crutchley and Minnick, 2012; Finkelstein and Hambrick, 1989), as well as manager-specific factors, such as discretion, human capital, social capital, and charisma (e.g., Ang et al., 2002; Bengtsson and Hand, 2011; Bizjak et al., 2011; Khan et al., 2005). In addition, political factors, such as board vigilance, remuneration committee, manager tenure, manager holdings, and manager family holdings, characterize the power and preferences of the board of directors and manager. Finkelstein and Hambrick (1989) argued that the interplay of power between board and executives would influence the compensation plan. Accordingly, performance-based determinants of managerial compensation should include managerial discretion, human capital, social capital,
The forms of payment for the managers can consist of financial compensation and non-financial compensation. The former includes salary, bonus, fringe benefits, stock options, pension contributions, deferred income, and long-term contingent compensation. The latter consists of leisure, the prestige associated with firm size, the working environment, and the extent of surveillance by stockholders (Finkelstein and Hambrick, 1989). This article exclusively considers cash compensation to simplify the research question (Shugan, 2002). The financial compensation schemes can be typically separated into two types of packages: behavior-oriented schemes (i.e., the base salary) and performance-oriented schemes (i.e., bonus and/or stock options) (Andergassen, 2008; Bizjak et al., 2011; Giummo, 2010; Huang et al., 2012).

In view of the role of managers as a hired hand, they should be compensated by base salary (Crutchley and Minnick, 2012; Milkovich and Newman, 1999), which is a so-called behavior-oriented compensation scheme. The base salary is the basic cash compensation that an employer pays for the work performed (Bronchetti, 2012; Giummo, 2010). But, in view of the role of managers as loose partners or professional managers, they expect other adequate incentives to reflect their continued efforts in this role (Milkovich and Newman, 1999). This kind of a supplemental compensation belongs to an outcome-oriented (Huang, 2012) or performance-oriented compensation scheme, which includes short- and long-term incentives. The former, which is defined as a bonus or profit-sharing bonus, is designed to fulfill the idea of attracting, retaining, and motivating executives (Milkovich and Newman, 1999). The long-term incentives, such as share options grants or stock options, are allocated under long-term incentive plans (Conyon et al., 2001). Its goal is to make managers to develop the feelings of ownership toward the business in the long run so that they perform as if they were the owner-managers (Milkovich and Newman, 1999). Based on these considerations, incentive managerial compensation contracts are associated with the behavior-oriented compensation schemes and performance-oriented compensation schemes.

3. The Model

Shugan (2002) noted that “Mathematics, as the language of science, allows interplay between empirical and theoretical research.” To evaluate the effectiveness of manager compensation, this study extends the concept of the BLSS model (Basu et al., 1985) to advance a model which considers a quota-based bonus. As assumed in the strategic delegation literature, managers are actually risk averse rather than risk neutral when environmental uncertainty is high (Sengul et al., 2012). An influential study, Vickers (1985), indicated the outcome of the game depended on the objectives of all the players and developed strategic delegation games to investigate the design of managerial incentive schemes. Several studies developed models to link internal agency logic to external competition arguments and suggested that the firm owner should use incentive compensation contracts for managers to reward them according
to their performance (e.g., Fershtman and Gneezy, 2001; Miller and Pazgal, 2001; van Witteloostuijn et al., 2007). For instance, Fershtman and Gneezy (2001) investigated the effects of strategic delegation and focused on the incentive compensation scheme in ultimatum games. Van Witteloostuijn et al. (2007) employed two-stage delegation games to explore the impact of disclosure of managerial compensation on consumer welfare and market competition.

In addition, the assumption of the BLSS model conforms to diminishing marginal utility for incomes and the utility of risk-averse individuals. The law of diminishing marginal utility is the prevalent theory in economics. Many studies have developed research models using nonlinear approaches based on the law of diminishing marginal utility and employed logarithms or negative exponential forms to reflect the nonlinear relationship (e.g., Ermini and Hendry, 2008; Horowitz et al., 2007). Accordingly, this study utilizes the concept of natural logarithms to develop a conceptual model which conforms to the law of diminishing marginal utility.

To develop the mathematical model, we first define necessary notation. First, this study assumes that the manager is risk averse and the degree of the $i$th manager’s risk aversion is $\rho_i$. In addition, the goal of the firm is supposed to maximize its expected profits, whereas the goal of the manager is maximize utility of income. A risk-averse manager wants to minimize the swings in his/her income. Thus, a risk-averse manager pursues the maximal utility of the certainty equivalent of income, denoted “C.E.”.

In addition, the law of diminishing marginal utility is the prevalent theory in the economic field. To conform to positive utility, this study assumes that the manager follows the law of diminishing marginal utility, and the utility of risk aversion (i.e., $U \geq 0$, $U' > 0$, and $U'' < 0$). The utility of the manager is a natural logarithmic function times a parameter, $\lambda$. Furthermore, each manager has an acceptable limitation of compensation, which is defined as the retainable income, $m$. The retainable income depends upon the job opportunity cost available from the most desirable alternative employment.

Third, although the determinants of compensation substitute as the surrogates of performance, each determinant possesses different influence on performance. According to the managerial compensation literature (e.g., Ang et al., 2002; Bengtsson and Hand, 2011; Bizjak et al., 2011; Conyon et al., 2001; Crutchley and Minnich, 2012; Finkelstein and Hambrick, 1989; Khan et al., 2005), the model of manager payment involves major sets of determinants of compensation (i.e., firm-specific market factors, manager-specific market factors, and political factors). Accordingly, performance-based determinants, $X_r$, include managerial discretion, human capital, social capital, charisma, tenure, manager family stock holdings, and firm size, performance, diversification, and risk. Therefore, the vectors of weights to take into account the impacts of the above determinants, $X_r$, is denoted $C_r$.

Fourth, Basu et al. (1985) employed three types of compensation plans, including straight salary, straight commissions, and combination of salary and commissions to discuss salesforce compensation plans. This study extends the Basu et al. (1985) ideal and further considers another compensation plan: a quota-based
bonus. In terms of the behavior-oriented compensation scheme, the firm offers the base salary, which is fixed in period \( t \). In terms of performance-oriented compensation scheme, the firm designs two forms of bonus to reward the managers involving the performance-based bonus, \( P \), and quota-based bonus, \( Q \). The performance-based bonus is awarded for the contribution of the manager to the firm’s performance and is an incentive to achieve maximal profits of the firm. In addition, the existing literature has expressed a strong interest in incentive compensation plans based on quota bonus in the competitive environment (e.g., Jain, 2012; Misra and Nair, 2011). Misra and Nair (2011) indicated that quotas are the essential component of compensation schedule. Jain (2012) suggested that one way to measure employee cumulative performance is to use multi-period quotas. The quota-based bonus is awarded when the total performance, \( D \), exceeds the pre-specified quota base, \( D \). Thus, the amount of the quota-based bonus is determined as \( nQ \) if the total performance is more than \( nQ \), but less than \( (n+1)Q \). For example, if the total performance of manager is 2.5 times of the quota base, the managers will earn a bonus of \( 2Q \).

3.1 Total Compensation and Certainty Equivalent

Based on Joseph and Thevaranjan (1998), this study assumes that the three forms of bonus follow the normal distribution. Specifically, the total cash compensation of the manager follows the normal distribution with mean \( S \) and variance \( (P^2 + Q^2)\sigma^2 \) for the sake of simplicity. That is \( S \sim N(S, (P^2 + Q^2)\sigma^2) \), where \( S = B + PCX + Q[D\mid D] \).

Because of risk aversion, the certainty equivalent should satisfy the following equation (Bierman and Fernandez, 1998): \( U(C.E.(S)) = E(U(S)) \), which implies that \( C.E. = E(S) + \rho \cdot \text{Var}(S) \), where \( \rho \) denotes the risk-aversion coefficient. Thus, the certainty equivalent of total compensation is equal to the sum of (i) the base salary, \( B \), (ii) the manager’s performance-based compensation, \( PCX \), (iii) the quota-based compensation, \( Q[D\mid D] \), and (iv) the product of the risk-aversion coefficient and the variance of compensation, \( \rho(P^2 + Q^2)*\sigma^2 \). Thus:

\[
C.E.(S) = S + \rho \left( P^2 + Q^2 \right) \sigma^2 = B + PCX + Q[D\mid D] + \rho \left( P^2 + Q^2 \right) \sigma^2.
\]

3.2 Analysis from the Firm’s Perspective

Given the assumptions of the model, we formulate the problem mathematically and derive the optimal compensation plan from the firm’s perspective. The firm’s objective is to maximize its expected profits, \( \pi \), from each manager. The firm is risk neutral in that it attempts to design an optimal compensation plan that can encourage each manager to contribute the most, \( R(S) \), while minimizing the cost of compensation, \( C(S) \). However, the compensation level should exceed the
individual acceptable level, $m_i$, so that the firm can retain the good managers. Thus, we can model this problem mathematically from the perspective of the firm as maximizing $\pi_i$ subject to $C.E. \geq m_i$. Replacing $C.E.$ with (1) and expanding the formula to derive the optimal compensation of the manager yields:

$$\max \left[ R(S_t) - C(S_t) \right]$$

subject to

$$B + P_i C, \bar{X} + Q_1 \left[ D_i / D_t \right] + \rho_i \left( P_i + Q_1 \right) \sigma^2 \geq m_i,$$

where $R(S_t)$ denotes the expected revenue in period $t$, which is equal to the sum of the contribution by the manager given the certain compensation and the contributions of others, $\beta_0$. The executives’ contribution is parallel to the performance-based determinants multiplied by marginal revenue, $\beta_1$, which represents the incremental revenue for each one unit contribution of the manager. Thus, the expected revenue can be expressed as:

$$R(S_t) = \beta_0 + \beta_1 \bar{X}.$$  

(4)

Also included in (2) is $C(S_t)$, which denotes the expected cost of compensation and equals the mean of total compensation, $\bar{S}$. That is:

$$C(S_t) = \bar{S} = B + P_i C, \bar{X} + Q_1 \left[ D_i / D_t \right].$$

(5)

In order to derive the optimal compensation plan, we substitute (4) and (5) into (2), and write the objective and subjective formulas (2) and (3) in Lagrangean form:

$$L_i(S_t) = \left[ \beta_0 + \beta_1 \bar{X} \right] - \left\{ B + P_i C, \bar{X} + Q_1 \left[ D_i / D_t \right] \right\}$$

$$+ \varepsilon \left\{ B + P_i C, \bar{X} + Q_1 \left[ D_i / D_t \right] + \rho_i \left( P_i + Q_1 \right) \sigma^2 - m_i \right\},$$

(6)

where $\varepsilon$ is the Lagrange multiplier.

Based on the Lagrange functions, the results of optimal base salary level and performance-based and quota-based bonus are shown in Table 1. The proof is given in Appendix A. From the firm’s perspective, the firm’s objective is to encourage managers to maximize the firm’s expected profit given the compensation level that the firm offers to managers should exceed the individual acceptable level for retaining good managers. To clarify and highlight the influence of each parameter, this study develops the following propositions based on the results of the optimal base salary level and bonus from the firm’s perspective.

**Proposition 1.** To maximize the firm’s profit, if retainable income of each manager or variance of each form of bonus increases, the base salary increases. In addition, if the risk-aversion coefficient, expected value of performance-based determinants, or expected value of quota-based determinants increases, the base salary decreases,
ceteris paribus.

**Proposition 2.** To maximize the firm’s profit, if the risk-aversion coefficient or expected value of performance-based determinants increases, the performance-based bonus increases. In addition, if variance of each form of bonus increases, the performance-based bonus decreases, ceteris paribus.

**Proposition 3.** To maximize the firm’s profit, if the risk-aversion coefficient, expected value of performance-based determinants, or expected value of quota-based determinants increases, the quota-based bonus increases. In addition, if variance of each form of bonus increases, the quota-based bonus decreases, ceteris paribus.

### 3.3 Analysis from the Manager’s Perspective

While the firm wants to maximize profits from managers, the managers want to maximize utility of certain income. Many studies have developed research models using nonlinear approaches based on the law of diminishing marginal utility (e.g., Mankiw, 2009). Several studies employ logarithms or negative exponential forms to reflect the nonlinear relationship (e.g., Ekstrom and Tysk, 2008). This study utilizes the concept of natural logarithms to develop a conceptual model that conforms to the law of diminishing marginal utility. Nevertheless, the maximum is not infinite in that the compensation is conditional upon their contribution. Namely, managers are endowed with the ability to negotiate only if they are expected to earn more than the profit base, $\pi_0$. For the sake of obtaining optimal compensation from the perspective of managers, the Lagrangean form can be written as follow:

$$
L_2(S_j) = \lambda \ln \left( \frac{\overline{B} + PC_j X + Q_i \left[ \frac{D_j}{D_0} \right] + \rho_i \left( P_i^2 + Q_i^2 \right) \sigma^2}{\left( \frac{D_j}{D_0} \right)} \right) + \delta \left( \beta \left( X \right) - \left( \frac{\overline{B} + PC_j X + Q_i \left[ \frac{D_j}{D_0} \right]}{\left( \frac{D_j}{D_0} \right)} \right) - \pi_0 \right).
$$

(7)

where $\delta$ is the Lagrange multiplier. The results are illustrated in Table 1. The proof is given in Appendix A. Based on the results of optimal base salary level and bonus from the manager’s perspective, this study develops the follow propositions.

**Proposition 4.** For maximizing the manager’s utilities, if the marginal revenue increases, the base salary increases. In addition, if expected value of performance-based determinants, expected value of quota-based determinants, or pre-specified profit base increases, the base salary decreases, ceteris paribus.

**Proposition 5.** To maximize the manager’s utility, if the expected value of performance-based determinants or the marginal revenue increases, the performance-based bonus increases. In addition, if expected value of quota-based determinants or the pre-specified profit base increases, the performance-based bonus decreases, ceteris paribus.
Proposition 6. To maximize the manager’s utility, if the expected value of quota-based determinants or the marginal revenue increases, the quota-based bonus increases. In addition, if the pre-specified profit base increases, the quota-based bonus decreases, ceteris paribus.

Table 1. The Optimal Compensation Plan

<table>
<thead>
<tr>
<th>Optimum</th>
<th>Firm’s Perspective</th>
<th>Manager’s Perspective</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base salary ((B^t))</td>
<td>(m_t - {P'C\vec{X} + Q[D/D_t]} + \rho_1(P'^t + Q'\sigma)} + {Q[D/D_t]} - \pi_t)</td>
<td>({\beta + \beta\vec{X}} - {P'C\vec{X}} + {Q[D/D_t]} - \pi_t)</td>
</tr>
<tr>
<td>Performance-based bonus ((P^t))</td>
<td>((e-1)(\vec{X}))</td>
<td>(\vec{X}(R(S)) - \pi_n - \vec{B})</td>
</tr>
<tr>
<td>Quota-based bonus ((Q^t))</td>
<td>(-2\rho_1\sigma)</td>
<td>(C, \vec{X} + [D/D_t])</td>
</tr>
<tr>
<td>Total compensation ((S^t))</td>
<td>(B^t + P'C\vec{X} + Q[D/D_t])</td>
<td>(B^t + P'C\vec{X} + Q[D/D_t])</td>
</tr>
</tbody>
</table>

3.4 Comparison between the Firm and Manager Perspectives

Since the optimal salary and bonus from the firm and manager perspectives are different, it is necessary to reconcile the two perspectives by identifying the negotiating range. The coefficient of each bonus is the same in each perspective, but the determinants and weights are different. Hence, apart from the base salary, we can obtain solutions by comparing magnitudes of coefficients with respect to the bonus. Since the salary and coefficient of each bonus from the manager’s perspective are always larger than those from the firm’s perspective, we derive that the base salary and each form of bonus that the managers desire are always larger than those the firm is willing to pay. Consequently, the negotiating range exists. For the base salary, the range is from \(m_t - \{P'C\vec{X} + Q[D/D_t]\} + \rho_1(P'^t + Q'\sigma)\} \) to \([\beta + \beta\vec{X}] - \{P'C\vec{X} + Q[D/D_t]\} - \pi_t\). For the performance-based bonus, the range is from \((e-1)(\vec{X})\) to \(\vec{X}(R(S)) - \pi_n - \vec{B}\). For the quota-based bonus, the range is from \((-2\rho_1\sigma\) to \(C, \vec{X} + [D/D_t]\). This result is summarized in the following proposition.

Proposition 7. Since the optimal salary and bonus from the firm and manager perspectives are different, a negotiating range exists and negotiation is necessary to obtain the equilibrium of base salary and each form of bonus.

3.5 The Dynamic Bargaining Game in Negotiation

Game theory has been generally accepted as a normative model of
decision-making and has been popularly applied to the fields of economics, sociology, and psychology. Since several decision problems can be thought of as games, game theorists have developed a large body of concepts and methods for analyzing games. The existing literature has expressed a strong interest in making strategic decisions based on game theory in the competitive environment (e.g., Bierman and Fernandez, 1998; Gans and Leigh, 2012; Sengul et al., 2012; van Witteloostuijn et al., 2007; Yang, 2010). Bierman and Fernandez (1998) suggested that game theory applied to bargaining begins to provide a mechanism to better understand the deals that would be struck. Bargaining theory has been seen a resurgent application in policy work (Gans and Leigh, 2012). Bargaining is an extremely important part of economic life and a great many decisions include bargaining in one form or another (Bierman and Fernandez, 1998). Van Witteloostuijn et al. (2007) further indicated that the majority of models in bargaining game tradition focused on wage bargaining. Accordingly, this study regards the negotiation process as a dynamic bargaining game. In this study, the firm’s objective is to encourage managers to maximize the firm’s expected profits and minimize the cost of compensation. In contrast, the managers pursue the maximal utility of certain income. Therefore, it is necessary to negotiate salary and bonus schemes to reconcile optimal compensation plans from the two perspectives.

Bargaining game researchers proposed that the equilibrium of the negotiation process depends on relative bargaining power of bargainers (e.g., Bierman and Fernandez, 1998; Mumcu, 2010; Sengul et al., 2012; Gans and Leigh, 2012; van Witteloostuijn et al., 2007). Bierman and Fernandez (1998) argued that the bargaining game model predicted that the gains two bargainers split from trade depended on who made the first offer, the number of rounds of offers, and the relative bargaining power of the two bargainers. Gans and Leigh (2012) provided a method to identify the level of relative bargaining power in bilateral negotiations. Van Witteloostuijn et al. (2007) focused on managerial compensation bargaining and evaluated the effectiveness of the relative bargaining power of the manager. Firm-specific human capital is the source of the worker’s increased bargaining power, and higher bargaining power can create a higher share of the surplus in the wage negotiation (Mumcu, 2010). These studies provided valuable contributions to the knowledge of bargaining power and suggested that if the bargaining process succeeded, the bargainer who had more relative bargaining power could obtain more benefit. For instance, van Witteloostuijn et al. (2007) indicated that the bargaining power of the managers could reduce firm profitability but enhance social welfare.

Based on these considerations, this study assumes the relative bargaining power of the firm over the manager in the negotiation process as benefits the firm can obtain. Let $\theta_1$ and $\theta_2$ denote the ratio of the negotiating range that the firm and the manager are willing to give up in the negotiation process, respectively. If $\theta_1$ is larger than $\theta_2$, the relative bargaining power of the firm is less than that of the manager. In other words, the firm is willing to give up a higher ratio of the negotiating range or is willing to pay more salary or bonuses in the negotiation process to provide more incentive in the managerial compensation contract. If $\theta_1$
equals \( \theta_1 \), both the firm and the manager are equally strong in the negotiation process. The firm and the manager take turns adjusting compensation offers until the equilibrium is offered and accepted (see Figure 1). From the manager’s perspective, he/she will gradually decrease the compensation demanded. Figure 2 represents the change in values of base salary for the manager in a dynamic bargaining process. Based on the optimal salary from both perspectives, the negotiating range of base salary lies between the optimal from the firm’s and the manager’s perspectives, which are expressed by the black line in Figure 2. Points to the left denote optima of base salary from the firm’s perspective, whereas points to the right denote optima of base salary from the manager’s perspective.

**Figure 1. The Game Tree for Bargaining as Manager Makes the First Move**

![Game Tree for Bargaining as Manager Makes the First Move](image1)

**Figure 2. Negotiation Range as Manager Makes the First Move**

![Negotiation Range as Manager Makes the First Move](image2)

The symbols of \( k \) and \( m \) represent the times that the firm and manager offer expected base salary in the negotiation process. If manager makes the first move and offers a value for the base salary (see Figures 1 and 2), \( V_{ij} \), the firm can accept or reject the offer. If firm accepts it, the game is over and the bargain is completed at the offered value. If, however, firm rejects the first offer, the negotiation range...
becomes the smaller one indicated by the dotted line marked $m=1$ in Figure 2.

The firm then makes a counter offer, $V_r$. Similarly, the manager can either accept or reject the counter offer. The negotiation will continue until the firm or manager accepts the other party’s offer.

If the firm accepts the value offered by the manager in the $m$th round, the equilibrium of base salary can be written as $B_r(MR) - \theta_r[B_r(MR) - B_r(Firm)] - \sum_{i=1}^{m-1} [(1-\theta_r)(1-\theta)]^{-i}$. On the other hand, if manager accepts the value offered by firm in the $k$ th round, the equilibrium of base salary can be written as $B_r(Firm) + \theta_r[B_r(MR) - B_r(Firm)] - \sum_{i=1}^{k-1} [(1-\theta_r)(1-\theta)]^{-i}$. This bargaining process can be displayed in the form of a game tree as shown in Figure 1. Likewise, this bargaining process can also apply to the negotiation of bonus. The equilibrium of bargaining process with respect to base salary and bonus are shown in Table 2.

Moreover, the equilibrium with respect to base salary and bonus, which the firm eventually accepts, will decrease when $m$ increases. Hence, the firm will desire to prolong the negotiation process in that the expected compensation that the manager offers decreases gradually. In contrast, the equilibrium with respect to base salary and bonus which the manager eventually accepts will increase as $k$ increases. Therefore, the manager will desire to prolong the negotiation process in that the compensation that the firm offers increases with the number of rounds. This is summarized in the following proposition.

Proposition 8. Since the equilibrium of base salary and bonus that the firm accepts is negatively related with the number of offers by the manager, the dominant strategy of the firm in the bargaining process is to accept the compensation offered by the manager in later rounds. Similarly, since the equilibrium of base salary and bonus that the manager accepts is positively related with the number of offers by the firm, the dominant strategy of the manager in the bargaining process is to accept the compensation offered by the firm in later rounds.

Table 2. The Equilibrium of Base Salary and Bonus when the Manager Makes the First Move

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>Firm accepts in $m$th round</th>
<th>Manager accepts in $k$th round</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base salary</td>
<td>$B_r(MR) - \theta_r[B_r(MR) - B_r(Firm)] - \sum_{i=1}^{m-1} [(1-\theta_r)(1-\theta)]^{-i}$</td>
<td>$B_r(Firm) + \theta_r[B_r(MR) - B_r(Firm)] - \sum_{i=1}^{k-1} [(1-\theta_r)(1-\theta)]^{-i}$</td>
</tr>
<tr>
<td>Performance-based</td>
<td>$P_r(MR) - \theta_r[P_r(MR) - P_r(Firm)] - \sum_{i=1}^{m-1} [(1-\theta_r)(1-\theta)]^{-i}$</td>
<td>$P_r(Firm) + \theta_r[P_r(MR) - P_r(Firm)] - \sum_{i=1}^{k-1} [(1-\theta_r)(1-\theta)]^{-i}$</td>
</tr>
<tr>
<td>bonus</td>
<td>$Q_r(MR) - \theta_r[Q_r(MR) - Q_r(Firm)] - \sum_{i=1}^{m-1} [(1-\theta_r)(1-\theta)]^{-i}$</td>
<td>$Q_r(Firm) + \theta_r[Q_r(MR) - Q_r(Firm)] - \sum_{i=1}^{k-1} [(1-\theta_r)(1-\theta)]^{-i}$</td>
</tr>
</tbody>
</table>

3.6 A Numerical Example

Several studies have proposed that numerical examples can help to illustrate or
Cheng-Feng Cheng

explain the results of mathematical model (e.g., Busby et al., 2012; Piccolo and Miklós-Thal, 2012; Yan and Sun, 2012). Accordingly, this study demonstrates the application of the optimal compensation model by referring to five cases (Cases A to E) to illustrate the contributions of this mathematical model. The numerical cases varied in terms of characteristics of firms and managers at random. Table 3 displays the values with regard to 10 performance-based determinants, $X_i$, the corresponding weights, $C_i$, and marginal revenue, $\beta$.

### Table 3. List of Example Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
<th>X4</th>
<th>X5</th>
<th>X6</th>
<th>X7</th>
<th>X8</th>
<th>X9</th>
<th>X10</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>$C_i$</td>
<td>0.05</td>
<td>0.08</td>
<td>0.11</td>
<td>0.09</td>
<td>0.13</td>
<td>0.15</td>
<td>0.12</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>$\beta_i$</td>
<td>0.08</td>
<td>0.11</td>
<td>0.14</td>
<td>0.12</td>
<td>0.16</td>
<td>0.18</td>
<td>0.15</td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>$\bar{X}$</td>
<td>7.0</td>
<td>7.5</td>
<td>6.5</td>
<td>6.2</td>
<td>6.6</td>
<td>4.0</td>
<td>6.7</td>
<td>7.6</td>
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</tr>
<tr>
<td></td>
<td>$C_i$</td>
<td>0.12</td>
<td>0.18</td>
<td>0.08</td>
<td>0.13</td>
<td>0.10</td>
<td>0.10</td>
<td>0.07</td>
<td>0.12</td>
<td>0.02</td>
</tr>
<tr>
<td>B</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_i$</td>
<td>0.14</td>
<td>0.20</td>
<td>0.10</td>
<td>0.15</td>
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<td>0.12</td>
<td>0.09</td>
<td>0.14</td>
<td>0.04</td>
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<tr>
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<td>0.10</td>
<td>0.15</td>
<td>0.12</td>
<td>0.12</td>
<td>0.09</td>
<td>0.14</td>
<td>0.04</td>
</tr>
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<td>0.12</td>
<td>0.05</td>
</tr>
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<td>0.18</td>
<td>0.10</td>
<td>0.20</td>
<td>0.16</td>
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<td>$C_i$</td>
<td>0.16</td>
<td>0.04</td>
<td>0.14</td>
<td>0.06</td>
<td>0.11</td>
<td>0.09</td>
<td>0.08</td>
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<td>0.07</td>
</tr>
<tr>
<td>D</td>
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<td>0.09</td>
<td>0.19</td>
<td>0.11</td>
<td>0.16</td>
<td>0.14</td>
<td>0.13</td>
<td>0.17</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>$\bar{X}$</td>
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<td>6.6</td>
<td>6.0</td>
<td>6.5</td>
<td>7.3</td>
<td>8.6</td>
<td>7.9</td>
<td>8.8</td>
<td>8.6</td>
</tr>
<tr>
<td></td>
<td>$C_i$</td>
<td>0.08</td>
<td>0.07</td>
<td>0.09</td>
<td>0.12</td>
<td>0.11</td>
<td>0.09</td>
<td>0.08</td>
<td>0.11</td>
<td>0.13</td>
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<td>E</td>
<td>$\beta_i$</td>
<td>0.12</td>
<td>0.11</td>
<td>0.13</td>
<td>0.16</td>
<td>0.15</td>
<td>0.14</td>
<td>0.13</td>
<td>0.15</td>
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<tr>
<td></td>
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<td>6.7</td>
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<td>5.6</td>
<td>6.5</td>
<td>5.0</td>
<td>7.0</td>
<td>7.8</td>
</tr>
</tbody>
</table>

Notes: $X_1$=Managerial discretion; $X_2$=Human Capital; $X_3$=Social capital; $X_4$=Charisma; $X_5$=Tenure; $X_6$=Stock holding; $X_7$=Firm size; $X_8$=Firm performance; $X_9$=Firm diversification; $X_{10}$=Firm risk. Scores of $X_i$ are in range from 0 to 10.

In addition, Table 4 displays the values of parameters in our model and the calculation of optimal values with regard to each type of compensation plan from both the firm and manager perspectives. The calculation of optimal values in the bottom of Table 4 indicates that both the firm and the manager in Case A prefer a low level of base salary and moderate level of bonus; both the firm and the manager in Case B prefer a low level of quota-based bonus; both the firm and the manager in Case C prefer a low level of base salary and low level of performance-based bonus; the firm in Case D prefers a low level of quota-based bonus but the manager prefers a high level of performance-based bonus; the firm in Case E prefers a high level of bonus. In terms of the base salary, the negotiation range in Case C is largest, whereas the negotiation range in Case D is smallest. In terms of the performance-based bonus, the negotiation range in Case D is largest, whereas the negotiation range in Case E is smallest. In terms of the quota-based bonus, the negotiation range in Case D is largest, whereas the negotiation range in Case E is smallest.
Table 4. The Values of Parameters in Example Cases

<table>
<thead>
<tr>
<th>Case</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_i$</td>
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<td>15</td>
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<tr>
<td>$m_i$</td>
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<td>20</td>
<td>25</td>
<td>30</td>
<td>35</td>
</tr>
<tr>
<td>$\varepsilon$</td>
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<td>0.89</td>
<td>0.92</td>
<td>0.95</td>
<td>0.82</td>
</tr>
<tr>
<td>$\rho_i$</td>
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<td>0.86</td>
<td>0.81</td>
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<td>0.92</td>
</tr>
<tr>
<td>$\sigma_i$</td>
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<td>0.40</td>
<td>0.30</td>
<td>0.40</td>
</tr>
<tr>
<td>$\pi_i$</td>
<td>10</td>
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<td>14</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>$B$</td>
<td>7.25</td>
<td>5.64</td>
<td>8.25</td>
<td>4.35</td>
<td>9.43</td>
</tr>
<tr>
<td>$\beta_i \bar{X}$</td>
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<td>9.14</td>
<td>9.75</td>
<td>11.27</td>
<td>9.43</td>
</tr>
<tr>
<td>$\theta_i$</td>
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<td>0.15</td>
<td>0.05</td>
<td>0.12</td>
<td>0.14</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>0.15</td>
<td>0.07</td>
<td>0.12</td>
<td>0.07</td>
<td>0.10</td>
</tr>
<tr>
<td>$k$</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$m$</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$C, \bar{X}$</td>
<td>6.54</td>
<td>7.63</td>
<td>7.09</td>
<td>7.50</td>
<td>6.60</td>
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<tr>
<td>$R^f (Firm)$</td>
<td>0.97</td>
<td>4.06</td>
<td>6.72</td>
<td>3.85</td>
<td>2.26</td>
</tr>
<tr>
<td>$B^f (MR)$</td>
<td>2.50</td>
<td>5.64</td>
<td>8.25</td>
<td>4.35</td>
<td>3.24</td>
</tr>
<tr>
<td>$P^f (Firm)$</td>
<td>1.47</td>
<td>1.57</td>
<td>0.95</td>
<td>1.04</td>
<td>1.97</td>
</tr>
<tr>
<td>$P^f (MR)$</td>
<td>1.66</td>
<td>2.19</td>
<td>1.21</td>
<td>2.74</td>
<td>2.11</td>
</tr>
<tr>
<td>$Q^f (Firm)$</td>
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<td>0.21</td>
<td>0.4</td>
<td>0.14</td>
<td>0.59</td>
</tr>
<tr>
<td>$Q^f (MR)$</td>
<td>0.51</td>
<td>0.29</td>
<td>0.51</td>
<td>0.37</td>
<td>0.64</td>
</tr>
</tbody>
</table>

Table 5. The Equilibrium of Base Salary and Bonus in Example Cases

<table>
<thead>
<tr>
<th>Compensation plans</th>
<th>Case</th>
<th>Firm accepts in $m$th round</th>
<th>Manager accepts in $k$th round</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base salary</td>
<td>A</td>
<td>1.961</td>
<td>1.199</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>5.373</td>
<td>4.455</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>7.913</td>
<td>6.891</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>4.286</td>
<td>3.989</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>3.142</td>
<td>2.610</td>
</tr>
<tr>
<td>Performance-based bonus</td>
<td>A</td>
<td>1.593</td>
<td>1.499</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>2.085</td>
<td>1.725</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1.153</td>
<td>0.979</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>2.524</td>
<td>1.512</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>2.096</td>
<td>2.020</td>
</tr>
<tr>
<td>Quota-based bonus</td>
<td>A</td>
<td>0.489</td>
<td>0.459</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.276</td>
<td>0.229</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>0.486</td>
<td>0.412</td>
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<td></td>
<td>D</td>
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<tr>
<td></td>
<td>E</td>
<td>0.635</td>
<td>0.608</td>
</tr>
</tbody>
</table>

The equilibrium compensation after negotiation is shown in Table 5. When the manager makes the first move in the negotiation process, and eventually the
manager accepts the values offered by the firm, the equilibrium of base salary and quota-based bonus are lowest relative to other negotiation types. In addition, the level of equilibrium of performance-based bonus is highest when the manager makes the first move and eventually the firm accepts the offered by the manager. The levels of equilibrium of base salary, performance-based bonus, and quota-based bonus are lower when the manager makes the first move and eventually the manager accepts the offer by the firm.

4. Conclusion

This article first integrates the determinants of executive compensation discussed in past research. We specify the determinants of compensation according to Finkelstein and Hambrick (1989) and add other determinants as suggested by later studies to our research model. We employ these determinants as a proxy for the evaluation of executive performance in our model. It is rational in that the determinants are related to not only the compensation but also to the executive contribution and effort. For example, managers will devote more effort to their job as the size or degree of diversification increases. In addition, the social capital and charisma can be viewed as the surrogates for managerial quality and leadership ability (Bizjak et al., 2011) that belong to the category of evaluating individual performance. Second, our optimal solution as developed in this study involves perspectives of the firm and the manager. The compensation model derives the optimal compensation plan from the two perspectives. Comparison of results indicates that a negotiating range exists. This range depends on values of the model parameters.

The model of this study is suitable for managers, which is different from the model for salesforces. The differences can be expounded in two ways. First, in the model of salesforce compensation, the level of sales is the determinant of total salary. However, the determinants of managers’ compensation are more complicated. Hence, this study specified two aspects to capture the complicated determinants of manager compensation, namely performance-based determinants, \( X', \) including managerial discretion, human capital, social capital, charisma, tenure, manager family stock holdings, and firm size, performance, diversification, and risk. In addition, we consider weights of the performance-based determinants, \( C, \) Thus, the product of \( C \) and \( X' \) represents total effect of individual performance on his/her own compensation. Second, since it is relatively difficult to replace managers than salespeople, managers possess more power to negotiate with the firm in terms of their compensation plan. Hence, this study established the range and process of negotiation particular to manager compensation plans.

Furthermore, this study extended the BLSS model of compensation and proposed an alternative model of compensation for managers. In addition, this study revised Joseth and Thevaranjan’s (1998) negative exponential utility function as a natural logarithmic utility function. In terms of the BLSS model, the authors employed three types of compensation plans including straight salary, straight commissions, and a combination of salary and commissions to discuss salesforce compensation plans. In addition, the BLSS model also considered the influences of scenario factors, which included uncertainty, marginal cost of production, expected
guaranteed utility, effectiveness of effort, and base sales level, on the structure of
total compensation plan. In this study, the compensation model was designed for
managers from perspectives of firms and managers. This study further established a
negotiation range to offer a reference point for both firms and managers when
negotiating compensation plans. In terms of the natural logarithmic utility function,
the equifinality was achieved in that the results derived from this utility function
approximate the results derived from the negative exponential utility function.
Additionally, this utility function meets the assumption of the BLSS model (i.e., the
law of diminishing marginal utility and the utility of risk aversion).

The propositions deduced from the model have several implications. In terms
of retainable income, a great deal of parameters, including bonus variance and the
degree of risk aversion, are positively associated with the base salary and negatively
with the bonus. This implies that the firm should raise the base salary in order to
encourage managers to take more account of safety and prefer certain contributions.
Additionally, the higher degree of dispersion of each bonus income signifies that the
firm or the environment is riskier. Increasing the base salary probably has the effect
of pacifying. The degree of risk aversion and the firm’s risk are closely bound up.
For example, if the firm tends to invest diversely, the firm risk will rise naturally. In
this case, if the firm hires the managers who are willing to undertake the risk, it
should diminish the base salary and raise the bonus in that the degree of risk
aversion is positively associated with the base salary. This course of action also
corresponds to the proposition that the base salary decreases with
performance-based determinants including the firm risk and diversification.

The pre-specified base, whether profit or quota, has a negative impact on the
quota-based bonus from perspectives of both the firm and the manager. It is
interesting that the quota-based bonus cannot motivate the managers to achieve
higher targets. Instead, reducing the quota-based bonus is advantageous to both the
firm and to the manager. However, the implications behind this statement involving
profit and quota base are different. In terms of the profit base, the firm can try to
reduce the quota-based bonus and base salary to lower costs. In terms of the quota
base, reducing the quota-based bonus may accompany increasing the base salary in
order to compensate the managers for bearing risk.

Although the optimal compensations from the firm and manager perspectives
are different, the negotiation process will reconcile the difference and achieve
equilibrium. Furthermore, the bargaining game provides strategies for negotiation.
According to the bargaining game, both parties will gradually move from their ideal
value in the negotiation process. In terms of the end of the negotiation process, both
the firm and the manager will be apt to accept the compensation the other party
offers in later rounds. This course of action implies another form of concession. If
the firm or manager wants to prolong the negotiation process, they have to make an
offer to encourage a counter offer. Note that the party will lose when the other party
offers. Therefore, making concessions in order to gain advantage is another strategy
for negotiation.

Essentially, our model provides several theoretical propositions. We also
suggest a few directions for future research. First, although we define the vectors of
weights to take into account the unequal impacts of determinants of compensation,
we do not investigate the weights by means of empirical analysis in order to focus on the central idea of this article. Second, the retainable income in our model is distinguishable from other studies in that this parameter varies across individuals. It is worthwhile to place more emphasis on this parameter.

Appendix A: Proofs of the Optimal Base Salary Level and Bonus

A1. Optimal base salary level and bonus from the firm’s perspective

Based on the Lagrangean from the firm’s perspective, we have:

\[
L_i(S_i) = \left[\beta_0 + \beta_i \bar{X}\right] - \left\{ \bar{B} + P_i C_i \bar{X} + Q_i \left[D_i/D_o\right] \right\} + \epsilon \left\{ \bar{B} + P_i C_i \bar{X} + Q_i \left[D_i/D_o\right] + \rho_i \left(P_i^2 + Q_i^2\right)\sigma^2 - m_i \right\}. \tag{A1}
\]

The optimal performance-based bonus \( P_i^* \) is:

\[
\frac{\partial L_i(S_i)}{\partial P_i} = 0 \Rightarrow -C_i \bar{X} + \epsilon C_i \bar{X} + 2P_i \epsilon \sigma^2 = 0
\]

\[
\Rightarrow P_i^* = \frac{(\epsilon - 1)C_i \bar{X}}{-2P_i \epsilon \sigma^2}. \tag{A2}
\]

Similarly, we obtain the optimum \( Q_i^* \) is:

\[
\frac{\partial L_i(S_i)}{\partial Q_i} = 0 \Rightarrow \left[D_i/D_o\right] + \epsilon \left[D_i/D_o\right] + 2\epsilon \rho_i Q_i \sigma^2 = 0
\]

\[
\Rightarrow Q_i^* = -\frac{\epsilon - 1 \left[D_i/D_o\right]}{-2\epsilon \rho_i \sigma^2}. \tag{A3}
\]

Substituting (A2) and (A3) into the subjective equation, we obtain the optimal base salary \( B^* \) as:

\[
\bar{B} + P_i C_i \bar{X} + Q_i \left[D_i/D_o\right] + \rho_i \left(P_i^2 + Q_i^2\right)\sigma^2 \geq m_i
\]

\[
\Rightarrow B^* = m_i - \left\{ P_i^* C_i \bar{X} + Q_i^* \left[D_i/D_o\right] + \rho_i \left(P_i^2 + Q_i^2\right)\sigma^2 \right\}. \tag{A4}
\]

A2. Optimal base salary level and bonus from the manager’s perspective

Based on the Lagrangean from the manager’s perspective, we have:

\[
L_i(S_i) = \lambda \ln \left[\bar{B} + P_i C_i \bar{X} + Q_i \left[D_i/D_o\right] + \rho_i \left(P_i^2 + Q_i^2\right)\sigma^2 \right] + \delta \left[\left(\beta_0 + \beta_i \bar{X}\right) - \left(\bar{B} + P_i C_i \bar{X} + Q_i \left[D_i/D_o\right]\right) - \pi_o \right]. \tag{A5}
\]
The optimum bonus levels are:

\[
\frac{\partial L_1(S_1)}{\partial P_t} = 0 \Rightarrow \frac{\lambda(C_t, \bar{X} + 2P_t \rho \sigma^2)}{B + P_tC_t, \bar{X} + Q_t[D_t/D_0] + \rho(P_t^2 + Q_t^2)\sigma^2} = \delta C_t, \bar{X} \quad (A6)
\]

\[
\frac{\partial L_2(S_2)}{\partial Q_t} = 0 \Rightarrow \frac{\lambda[D_t/D_0] + 2Q_t \rho \sigma^2}{B + P_tC_t, \bar{X} + Q_t[D_t/D_0] + \rho(P_t^2 + Q_t^2)\sigma^2} = \delta[D_t/D_0] \quad (A7)
\]

To derive solutions, dividing (A6) by (A7) yields:

\[
\frac{(A6)}{(A7)} \Rightarrow \frac{C_t, \bar{X} + 2P_t \rho \sigma^2}{[D_t/D_0] + 2Q_t \rho \sigma^2} = \frac{C_t, \bar{X}}{[D_t/D_0]}
\]

\[
\Rightarrow C_t, \bar{X}[D_t/D_0] + 2P_t \rho \sigma^2[D_t/D_0] = C_t, \bar{X}[D_t/D_0] + 2Q_t \rho \sigma^2C_t, \bar{X}
\]

\[
\Rightarrow P_t[D_t/D_0] = Q_tC_t, \bar{X} \quad (A8)
\]

Substituting (A8) into the subjective equation we obtain:

\[
R(S_t) - C(S_t) = \pi_0 \Rightarrow B + P_tC_t, \bar{X} + \left[D_t/D_0\right] P_t = R(S_t) - \pi_0
\]

\[
\Rightarrow P_t = \frac{C_t, \bar{X}[R(S_t) - \pi_0 - B]}{(C_t, \bar{X})^2 + [D_t/D_0]^2} \quad (A9)
\]

Equation (A9) is the optimal performance-based bonus. Combining (A9) and (A8), we obtain the optimal quota-based bonus:

\[
Q_t = \frac{[D_t/D_0]}{C_t, \bar{X}} P_t = \frac{[D_t/D_0][R(S_t) - \pi_0 - B]}{(C_t, \bar{X})^2 + [D_t/D_0]^2} \quad (A10)
\]

Substituting (A9) and (A10) into the subjective equation, we obtain the optimal base salary \( B_t' \):

\[
B_t' = [\beta_t + \beta_t, \bar{X}] - [P_tC_t, \bar{X} + Q_t[D_t/D_0]] - \pi_0 \quad (A11)
\]

Notes

1. Based on Joseph and Thevaranjan (1998), this study assumes that the performance-based compensation follows the normal distribution with mean \( PC_t, \bar{X} \) and variance \( (P_t^2)\sigma^2 \). In addition, the quota-based compensation follows the normal distribution with mean \( Q[D_t/D_0] \) and variance \( (Q_t^2)\sigma^2 \).
2. According to Bierman and Fernandez (1998), the certainty equivalent can be expressed as a simple function of the expected payoff and a measure of its risk.
3. The BLSS model identified several structural parameters that affect the compensation plan, and Joseph and Thevaranjan (1998) developed the compensation plan based on both incentive and
monitoring perspectives. To elaborate the contribution of the optimal compensation plan, this study applies game theory to optimize the manager compensation plan from perspectives of both the firm and the manager.

References