A Note on Redistributive Taxation, Labor Supply, and National Income

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Abstract
This paper shows that redistributive taxation can raise per capita income provided that labor supply is sufficiently backward-bending at higher wage rates. Moreover, we study general equilibrium effects regarding work incentives for less able individuals. Finally, we show that optimal taxation typically requires per capita income to decrease with higher taxation at the margin.

Key words: backward-bending labor supply; linear income tax; national income
JEL classification: H31; H21

1. Introduction

According to conventional wisdom, redistribution is costly in terms of losses in gross domestic product per capita. In particular, it is argued that redistributive taxation creates disincentive effects on the labor (or effort) supply of both rich and poor individuals.

However, it has been convincingly discussed that potentially income-enhancing effects of redistribution policies arise at least in developing countries. First, when malnutrition is a severe problem, redistribution towards the poor can obviously raise average productivity (e.g., Dasgupta, 1993). Second, if there are credit market imperfections such that poor households cannot borrow to finance higher education, redistribution can raise the human capital stock of an economy (e.g., Bénabou, 1996).

In this paper we examine whether it is possible for (distortionary) redistributive income taxation to raise per capita income even in an ideal framework of a perfect market economy (i.e., in the standard labor-leisure choice model). Answering this question is important, since national income per head is undoubtedly the most widely used (although a very crude) indicator for evaluating the performance or...
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...even social welfare in real economies. Moreover, the analysis takes up the debate about disincentive effects of redistribution for less able individuals and examines the relationship between optimal linear income taxation and per capita income.

It is shown that distortionary redistribution can indeed raise per capita income, provided that labor supply behavior is backward-bending at higher wage rates. Moreover, labor supply behavior at higher wages has a feedback effect on work incentives for less able individuals by affecting transfer income. For instance, under a non-negative relationship between redistributive taxation and national income, labor supply for low-wage individuals is unambiguously decreasing with higher taxation. However, this no longer holds if the impact of an increase in taxation on national income is negative. Finally, it is shown that, typically, the socially optimal linear income tax rate should have a negative impact on per capita income at the margin.

The paper is organized as follows. Section 2 sets up a simple model. Section 3 derives the individual labor (or effort) supply choice, whereas Section 4 analyzes the equilibrium and discusses the results. To partially justify the crucial role of backward-bending labor supply behavior at higher wage rates, Section 5 briefly reviews empirical evidence on a negative (uncompensated) wage elasticity of labor supply. The last section concludes.

2. The Model

Consider a set $\mathcal{I} = \{1, \ldots, n\}$ of individuals, each indexed by $i$ with earning abilities $w_i > 0$ and identical time endowments $\tilde{l}$ (which can also be interpreted as “maximum” effort level). There are no market imperfections, which implies full employment of labor. Following the literature on optimal income taxation (e.g., Mirrlees, 1971; Sheshinski, 1972), let wage rates be equal to abilities $w_i$. In a perfect market economy with one homogenous consumption good, this is consistent with a constant-returns-to-scale production technology in which different types of labor are perfectly substitutable (i.e., the production function is given by $F(l_1, \ldots, l_n) = \sum w_i l_i$).

The preferences of the $i$th individual are represented by some twice continuously differentiable utility function $u(c_i, l_i)$ with $u'_1 > 0$, $u'_2 < 0$, $u'_{11} < 0$, $u'_{22} < 0$, and $u'_{12} = 0$ and where $c_i$ denotes the consumption level and $l_i$ is the labor (or effort) supply of individual $i$. Shortly we will see that these assumptions ensure both strict quasiconcavity of $u'$ and normality of leisure ($I - l_i$).

Let $Y = (1/n)\sum_i y_i$ denote per capita (labor) income, where $y_i = w_i l_i$. Individuals face a co-linear tax scheme with an income tax rate $t \in (0, 1)$ and a (per capita) lump-sum transfer $T$. Thus, $c_i = (1-t)w_i l_i + T$. The government budget is balanced (i.e., $T = tY$). Thus, an individual with $y_i < Y$ ( $y_i > Y$) receives a positive (negative) net transfer.
3. Individual Choice

Taking \( t \) and \( T \) (hence, \( Y = T / t \)) as given, optimal labor supply \( \bar{l}_i \) of individual \( i \) equals

\[
\bar{l}_i = \varphi_i(t, Y, w_i) := \arg \max_{l_i \in [0, T]} u'(l_i(1-t)w_i + tY + l_i). \tag{1}
\]

Here, we assume that \( \bar{l}_i \) is an interior solution, given by the first-order condition

\[
F'(t, w_i, Y; \bar{l}_i) = 0, \tag{2}
\]

That is, the marginal rate of substitution \( MRS^i := -u'_2 / u'_1 \) must be equal to the net (or after-tax) wage rate \( w_i(1-t) \). Note that

\[
\frac{\partial F'(t, w_i, Y; \bar{l}_i)}{\partial l_i} = u'_{11}(1-t)^2 w_i^2 + 2u'_{12}(1-t)w_i + u'_{22} < 0, \tag{3}
\]

(i.e., the second-order condition for the maximization problem (1) is fulfilled). Thus, \( MRS^i \) is increasing in \( l_i \). Now define

\[
\varepsilon_i := \left( -\frac{l_i}{u'_1} \right) \left( \frac{\partial u'_1(l_i(1-t)w_i + tY + l_i)}{\partial l_i} \right)_{l_i=\bar{l}_i}, \tag{4}
\]

as the elasticity of marginal utility from consumption \( u'_1 \) with respect to labor supply \( l_i \) evaluated at the optimal choice \( l_i = \bar{l}_i \). Note that \( \varepsilon_i > 0 \), as \( u'_{11} < 0 \), \( u'_{12} \leq 0 \), and \( t < 1 \).

**Lemma 1:** In an interior solution for the individual labor supply decision, we have (i) \( \frac{\partial \bar{l}_i}{\partial Y} < 0 \), (ii) \( \frac{\partial \bar{l}_i}{\partial w_i} < 0 < \varepsilon_i = 1 \), and (iii) \( \frac{\partial \bar{l}_i}{\partial t} < 0 \) if \( Y < 1 \), where \( \bar{y}_i = w_i \bar{l}_i \).

**Proof:** First, note that \( MRS^i = -u'_2 / u'_1 \) is increasing in \( Y \) due to \( t < 0 \), \( u'_{11} < 0 \), and \( u'_{12} \leq 0 \). Also recall that \( MRS^i \) is increasing in \( l_i \). Thus, if \( Y \) increases, the optimal (amount of) labor supply must decrease in order to maintain the equality given by \( MRS^i = w_i(1-t) \). This proves part (i). Parts (ii) and (iii) are proven by applying the implicit function theorem to \( F'(t, w_i, Y; \bar{l}_i) = 0 \). We have

\[
\text{sign} \left( \frac{\partial \bar{l}_i}{\partial x} \right) = \text{sign} \left( \frac{\partial F'(t, w_i, Y; \bar{l}_i)}{\partial x} \right), \tag{5}
\]

for \( x = t, w_i \). From (2), it is straightforward to calculate that
\[ \frac{\partial F^i()}{\partial w_i} = (u^i_{21}(1-t)w_i + u^i_{12}(1-t)w_i + u^i_1(1-t)), \quad (6a) \]

\[ \frac{\partial F^t()}{\partial t} = -(w_i l_i - Y) (u^i_{11}(1-t)w_i + u^i_{21}l_i) - u^i_1 w_i. \quad (7a) \]

From (4), we obtain \( \varepsilon_i = -(u^i_{11}(1-t)w_i + u^i_{21}l_i) (w_i / l_i) \). Using this expression and evaluating \( \frac{\partial F^i()}{\partial w_i} \) and \( \frac{\partial F^t()}{\partial t} \) at \( l_i = \bar{l}_i \), (6a) and (7a) imply

\[ \frac{\partial F^i()}{\partial w_i} \bigg|_{l_i=\bar{l}_i} = -u^i_1 (1-t)(\varepsilon_i - 1), \quad (6b) \]

\[ \frac{\partial F^t()}{\partial t} \bigg|_{l_i=\bar{l}_i} = u^i_1 w_i [(1-Y[Y])e_i - 1]. \quad (7b) \]

Finally, recalling (5) and \( 0 < t < 1 \), we prove parts (ii) and (iii), respectively.

First, note that an increase in per capita income \( Y \) means that transfer income \( T = tY \) rises, inducing a pure income effect. Thus, according to part (i) of Lemma 1, leisure is a normal good. According to part (ii), for a given transfer income \( T \), individual labor supply is a non-decreasing function of the wage rate \( w_i \) if \( \varepsilon_i \) is sufficiently low (i.e., if \( \varepsilon_i \leq 1 \)). In contrast, if \( \varepsilon_i > 1 \), optimal labor supply \( \bar{l}_i \) is backward-bending as a function of \( w_i \). In this case, the income effect of a higher \( w_i \) on labor supply outweighs the substitution effect. Ultimately, the shape of the labor supply curve in \( l_i - w_i \) space is an empirical question. Section 5 gives a brief account of the empirical evidence on the wage elasticity of labor supply.

To understand part (iii), note that an increase in the tax rate \( t \) has two effects on \( \bar{l}_i \). First, the net wage decreases, which induces the opposite effect as an increase in \( w_i \). Second, however, transfer income \( T = tY \) increases, which reduces \( \bar{l}_i \) by an income effect. Thus, \( \varepsilon_i > 1 \) is not sufficient for the optimal labor supply \( \bar{l}_i \) to increase with \( t \). In fact, for individuals with \( \bar{Y}_i \leq Y \), \( \bar{l}_i \) decreases unambiguously with \( t \). In contrast, if the labor supply curve in \( l_i - w_i \) space is backward-bending (i.e., if \( \varepsilon_i > 1 \)), an increase in the tax rate \( t \) may have a positive net effect on \( \bar{l}_i \) for high-income individuals (i.e., if \( \bar{Y}_i > Y \)). In equilibrium, of course, per capita income \( Y \) (and, hence, transfer income \( T \)) is endogenous, which is the subject of the following analysis.

4. Equilibrium

4.1. Redistributive Taxation and Per Capita Income

The first part of this section examines the impact of redistributive taxation on equilibrium per capita income, which is denoted by \( Y^* \). Note that \( Y^* \) and the equilibrium individual labor supply levels \( \bar{l}_i \), \( i \in \mathcal{I} \), respectively, are simultane-
ously given by (the \( n + 1 \) equations)
\[
I_i^* = \xi_i(t, Y^*, w_i), \quad i \in \mathcal{I},
\]
\[
Y^* = \sum_i w_i I_i^*.
\]
Conditions (8) and (9) imply that \( Y^* \) is implicitly defined by
\[
\sum_i w_i \xi_i(t, Y^*, w_i) - n Y^* = 0,
\]
as a function of \( t \), \( n \), and \( w_1, ..., w_n \). Note from (8) and (9) that
\[
\sum_i w_i \frac{\partial \xi_i(t, Y^*, w_i)}{\partial t} \text{ is the marginal impact of an increase in } t \text{ on aggregate income when neglecting general equilibrium effects of taxation on } Y^*.
\]
Regarding the relationship between per capita income \( Y^* \) and the tax rate \( t \), the following is implied.

**Proposition 1:** The impact of an increase in the tax rate \( t \) on \( Y^* \) is positive (negative) iff \( \sum_i w_i \frac{\partial \xi_i(t, Y^*, w_i)}{\partial t} > ( <) 0 \).

**Proof:** Applying the implicit function theorem to (10), we have
\[
\frac{\partial Y^*}{\partial t} = - \frac{\sum_i w_i \frac{\partial \xi_i(t, Y^*, w_i)}{\partial t}}{\sum_i w_i (\frac{\partial \xi_i(t, Y, w_i)}{\partial Y})_{Y=Y^*} - n}.
\]
Recall that \( \frac{\partial \xi_i(t, Y, w_i)}{\partial Y} < 0 \) for all \( i \), according to part (i) of Lemma 1. Thus, the denominator of the right-hand side of (11) is negative. Hence, the result directly follows from the numerator in (11).

In order to focus the discussion, it is supposed in the following that preferences are such that equilibrium pre-tax income \( Y_i^* = w_i I_i^* \) is increasing in the wage rate \( w_i \) for all individuals (i.e., \( \frac{\partial I_i^*}{\partial w_i}(w_i, I_i^*) > -1 \) for all \( i \in \mathcal{I} \)). In other words, envisage a monotonic relationship between an individual’s earning ability and individual wage income. For instance, it can be shown (available upon request) that this regularity assumption is fulfilled for the case of \( u'(c_i, l_i) = \ln(c_i - \tau') - (l_i)^{\alpha'} \) with \( \tau' \geq 0 \) and \( \alpha' > 1 \). This specification meets the assumptions made in Section 2 and allows for backward-bending labor supply behavior.

Define the set \( \mathcal{J} := \{i \in \mathcal{I}: Y_i^* > Y^* \} \) of (relatively able) individuals with equilibrium income above the average. According to Proposition 1, for \( \partial Y^*/\partial t > 0 \) to hold, it is necessary that \( \partial I_i^*/\partial t > 0 \) for at least some \( i \in \mathcal{I} \). According to parts (ii) and (iii) of Lemma 1, this can only be the case for relatively able individuals \( i \in \mathcal{J} \), requiring that their labor supply curve (in \( l_i - w_i \) space) is backward-bending. As argued in Section 5 below, this is not an implausible case. It is also related to widely discussed phenomena of material saturation of the rich (e.g., Saint-Paul, 2001).
4.2 (Dis)Incentive Effects of Redistribution

Policy makers are often concerned with disincentive effects of redistribution towards the poor for individuals with low earning abilities. This subsection addresses this issue in light of the preceding analysis. In fact, according to part (iii) of Lemma 1, \( I_i \) unambiguously declines with \( t \) for low-income individuals, for a given per capita income \( Y \). In order to analyze whether this is still true in equilibrium, we distinguish the cases \( \partial Y^* / \partial t \geq 0 \) and "conventional wisdom" \( \partial Y^* / \partial t < 0 \). The following result emerges.

**Proposition 2:** For all \( i \in \mathbb{N} \setminus \hat{\mathbb{N}} \) (i.e., if \( Y_i^* \leq Y^* \)), if \( \partial Y^* / \partial t \geq 0 \), then an increase in the tax rate \( t \) reduces equilibrium labor supply \( I_i^* \). If \( \partial Y^* / \partial t < 0 \), the impact of an increase in \( t \) on \( I_i^* \) is ambiguous for all \( i \in \mathbb{N} \).

**Proof:** Note that the total impact of an increase in \( t \) on \( I_i^* \) is given by

\[
\frac{dI_i^*}{dt} = \frac{\partial I_i^*}{\partial t} + \left( \frac{\partial^2 I_i^*}{\partial t \partial Y} \right) \frac{\partial Y^*}{\partial t}.
\]

According to part (iii) of Lemma 1, \( \frac{\partial I_i^*}{\partial t} \leq 0 \) if \( Y_i^* \leq Y^* \). Moreover, \( \frac{\partial^2 I_i^*}{\partial t \partial Y} < 0 \), according to part (i) of Lemma 1. Thus, \( \frac{dI_i^*}{dt} < 0 \) if \( \partial Y^* / \partial t < 0 \), but not necessarily if \( \partial Y^* / \partial t \geq 0 \). This concludes the proof.

According to Proposition 2, the possibility that \( \partial Y^* / \partial t \geq 0 \) is consistent with the widespread notion that redistribution creates disincentive effects for less able individuals. However, in an equilibrium having \( \partial Y^* / \partial t < 0 \), this result may no longer hold. The reason is that a decrease in \( Y^* \) has an income effect, related to a decline in the equilibrium transfer \( T^* = tY^* \). This income effect induces an increase in work effort, rendering the total effect of \( t \) on labor supply ambiguous. Thus, the concerns of policy makers that redistribution reduces both national income and effort supply for low-skilled workers may be in conflict with each other.

4.3 Socially Optimal Redistribution

Would a social planner without a "bias" towards rich individuals \( i \in \hat{\mathbb{N}} \) choose the optimal linear income tax (under a balanced budget requirement) in a range where (at the margin) a higher tax rate \( t \) increases or decreases equilibrium per capita income \( Y^* \)? To answer this question, consider an increasing, concave, and twice continuously differentiable social welfare function \( W : \mathbb{R}^n \rightarrow \mathbb{R} \). The optimal tax rate \( t^* \) solves

\[
\max W\left( u^i((1-t)w_1I_1^* + tY^* + I_1^*), ..., u^i((1-t)w_nI_n^* + tY^* + I_n^*) \right).
\]

Using (by the envelope theorem) the first-order condition \( u_i^iw_i(1-t) + u_i^k = 0 \)
from the individual choice problem, the first-order condition of the social planning problem (13) can be written as (denoting $W_i = \partial W_i / \partial u_i^i$)

$$\frac{\partial Y^*}{\partial t} \bigg|_{t = t^*} = -\left( \sum_{i} W_i u_i^i (Y^* - y_i^* ) \right) \left( \sum_{i} W_i u_i^i \right)_{t = t^*}.$$  (14)

Note that for individuals with $y_i^* < Y^*$, the marginal utility of consumption $u_i^i$ is relatively high. Thus, the term $u_i^i (Y^* - y_i^* )$ is positive and relatively high. The opposite holds for relatively able individuals $i \in \hat{X}$. Hence, for instance, under a utilitarian social welfare function (i.e., if $1_i W_i$ for all $i$), the impact of an increase in $t$ on $Y^*$, evaluated at the optimal tax rate $t^*$, should be negative. The same holds whenever $W_i$ is “not too high” for rich individuals $i \in \hat{X}$ relative to less able individuals (“no bias to the rich”). Hence, generally, the following can be concluded from (14).

**Proposition 3:** A social planner without bias towards able individuals would choose a linear income tax with a balanced budget such that $\frac{\partial Y^*}{\partial t} \bigg|_{t = t^*} < 0$.

Thus, from a social point of view, it is desirable to redistribute income in a way which reduces per capita income in the economy under consideration. This has an important policy implication. Even though it may be possible to redistribute income such that national income does not decrease, it is not socially optimal to do so, provided that the social planner has no bias to the rich.

5. Empirical Relevance of Backward-Bending Labor Supply

As argued in the preceding section, a necessary condition for the result that equilibrium per capita income increases with the tax rate is that individual labor supply as a function of the wage rate is backward-bending at higher wages (assuming a monotonic relationship between an individual’s earning ability and individual wage income in equilibrium). This section briefly reviews empirical evidence on the shape of the labor supply curve.

In their well-known survey articles, Hausman (1985) and Pencavel (1986) come to quite different conclusions regarding the sign of the (uncompensated) wage elasticity of male labor supply. Whereas Pencavel (1986) reports that estimates of the wage elasticity tend to lie in the range between $-0.17$ and $-0.08$, providing some support for backward-bending labor supply, most studies reviewed in Hausman (1985) find a small and mostly positive wage elasticity. In their now famous study, MaCurdy et al. (1990) try to reconcile this conflicting evidence. The studies covered by Hausman (1985) explicitly account for taxation by linearizing tax schedules piecewise. Contrary to the notion that these studies are more relevant for the analysis of tax effects on labor supply, MaCurdy et al. (1990) argue that estimates derived by use of a piecewise-linear econometric methodology implicitly restrict parameter values such that both substitution and income effects are driven to be zero. As these
restrictions cannot be justified by economic theory, they conclude that the small (non-negative) estimates of wage elasticities “are essentially imposed by procedure” (MaCurdy et al., 1990, p. 417; for further discussion, also see Blundell and MaCurdy, 1999).

Estimates of uncompensated wage elasticities of female labor supply (mostly analyzing behavioral responses of married women) tend to be mostly positive (e.g., Killingsworth and Heckman, 1986; Blundell et al., 1998), although heavily depending on the specification of the econometric model (e.g., Mroz, 1987). For instance, Blundell et al. (1993) and Puhani (1995) find negative wage elasticities for married women in France and Poland, respectively.

The theoretical model presented in this paper highlights the importance of backward-bending labor supply behavior of relatively able individuals for the impact of redistributive taxation on per capita income. Thus, a proper empirical account of the theoretical predictions derived in this paper requires a specification of an empirical model which allows for non-linearity of the labor supply curve. In fact, as pointed out by MaCurdy et al. (1990, p. 466), “this specification must capture relevant behavioral features, such as a backward-bending property which evidence suggests is a factor in determining men’s hours of work”. A recent paper by Jang (1998) has proceeded along these lines, providing nonparametric estimates of the labor supply curve which explicitly allows for backward-bending behavior. His findings strongly support backward-bending labor supply for both single females and wives with working husbands in the US at higher hourly wage rates, whereas wage elasticities are positive at lower wages. There is also weak evidence for a negative wage elasticity of married men, with and without working wives, at higher hourly wages.

In sum, the empirical evidence and debate on negative (uncompensated) wage elasticities of labor supply is still unsettled, partially due to the choice of econometric models. However, it seems fair to conclude that backward-bending behavior of labor supply is not merely a theoretical possibility, but a frequently reported finding.

6. Conclusion

Over the last three decades, a large literature on optimal income taxation has developed. Unfortunately, however, general results regarding the optimal tax system are rare. Even for the linear income tax studied in this note, results crucially depend on the specification of individual preferences and the social welfare function (e.g., Atkinson and Stiglitz, 1980).

Pragmatically, the by far most popular indicator for the performance and social welfare of an economy used by policy makers around the world is national income (per capita). This note has focused on the impact of redistributive (linear) income taxation on this indicator. It has been shown that, even in a perfect market economy with endogenous labor supply, redistributive taxation does not necessarily reduce per capita income. Rather, the analysis has highlighted the importance of backward-bending labor supply behavior at higher wage rates for the relationship be-
tween redistribution and per capita income. Moreover, it has been shown that redistributing to less able individuals unambiguously creates disincentive effects to the poor if national income indeed rises with a higher tax rate. However, if it falls, as suggested by conventional wisdom, individuals with low earning abilities may increase labor supply due to the reduction in transfer income. Finally, it has been shown that, if the social welfare function is not biased towards relatively able individuals, the optimal linear income tax rate is such that national income per head decreases at the margin.

References

