Tax Evasion and Monopoly Output Decisions Revisited: Strategic Firm Behavior

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Abstract
This note reexamines the model of tax evasion of the monopolistic firm with profit taxes by incorporating the firm’s strategic behavior for tax avoidance. It is shown that under certain conditions, the monopolist’s decisions on output and expenditure are no longer separable from the evasion decision and there is a trade-off between production efficiency and cost efficiency. We then derive the optimal profit tax rate to investigate some properties of profit taxation.

Key words: tax evasion; strategic expenditure; optimal profit taxation

JEL classification: D8; H26

1. Introduction

The subject of tax avoidance has been an area of ongoing debate since the seminal paper of Allingham and Sandmo (1972). One vein of economic literature on tax evasion has been concerned with taxes for monopolistic firms. Marrelli (1984) first examined the firm’s decisions about the amount of tax evaded and the quantity produced to avoid an ad valorem tax by manipulating under-reporting revenue. A pioneering study on the profit tax evasion was investigated by Kreutzer and Lee (1986). They used a simple model of a monopolistic firm that can reduce its tax liability by under-reporting profits through an overstatement of production costs provided that the actual unreported costs are undetectable by the authorities. They explored the possibility of using a profit tax to reduce monopoly distortion and showed that the tax-evading activity, in the absence of a penalty, will induce the firm to expand output beyond the no-tax level. This is an important result because it is well-known that in a certain world, a tax on profits will have no effect on the output decisions of a profit-maximizing monopolistic firm.

Wang and Conant (1988) added realism by formulating an uncertainty model under the assumption that the probability of detection and the penalty rate are

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exogenously fixed and showed that a firm’s optimal production will be determined independent of the cost-overstatement decision, challenging the results of Kreutzer and Lee. Kreutzer and Lee (1988) replied that the Wang-Conant approach biased the solution toward reliance on the cost overstatement factor by assuming that the probability of detecting tax evasion is independent of the cost overstatement factor. Again, Yaniv (1996) extended the Wang-Conant model under the assumption that both the probability of detection and the penalty rate increase with the amount of understated profits and concluded that profit taxes are neutral with respect to a monopolist’s profit-maximizing rate of output. Moreover, when the monopolist decides on the amount of cost overstatement rather than on the proportion of cost statement, it was shown that the separability of the output decision holds even if evasion is not optimally chosen. This was a surprising result since symmetric separability between the labor-supply decision and the tax-evasion decision does not arise in individual tax evasion. However, Kim (1997) modified the Kreutzer-Lee model under the assumption that the probability of detection increases with the cost overstatement factor and supported the Kreutzer-Lee argument that profit taxes will increase the monopolist’s profit-maximizing rate of output. He also compared the social welfare due to tax imposition on both a uniform and a two-part pricing scheme.

In this note, we reexamine the monopolist’s decisions on output and tax evasion under the more realistic situation where the potential for moral hazard of the strategic firm behavior exists. As indicated by Kreutzer and Lee, we explicitly incorporate the possibility that actual production costs and wasteful expenditures are not differentiated by the authorities under profit regulation. Under the assumption that the probability of under-reporting profits being detected is a function of the cost overstatement factor, it is shown that the monopolist’s decisions concerning output and expenditures are no longer separable from the evasion decision. This implies that the conventional result under certainty that profit taxes are neutral with respect to the monopolist’s profit-maximizing output level does not hold. It is also found that profit taxation no longer results in production at minimum cost. Therefore, there is a trade-off between production efficiency and cost efficiency in the context of profit taxation. Finally, from the normative welfare perspective, we derive and analyze the optimal profit tax rate which maximizes the social welfare.

The remainder of this paper is organized as follows. In Section 2, we examine the basic model in which the monopolistic firm can engage in strategic behavior of wasteful expenditures when the probability of detection and the penalty rate are exogenously fixed. In Section 3, we extend the basic model into the case where the probability of detection and the penalty rate are functions of the over-reporting factor. We then investigate the monopolist’s decisions on output and strategic expenditures for tax avoidance and derive the optimal profit tax rate. Concluding remarks are provided in the final section.
2. The Basic Model

Consider the decision-making problem of a monopoly firm facing a profit tax rate \( t \), where \( 0 < t < 1 \). The firm’s output, denoted \( Q \), is defined such that the inverse market demand function is \( P(Q) \), where \( P'(Q) < 0 \), and the firm’s production function is \( C(Q) \), where \( C'(Q) > 0 \) and \( C''(Q) \geq 0 \).

We consider the strategic possibilities of the monopolistic firm under profit taxation, as indicated in Sappington (1980), Blackmon (1992), Laffont and Tirole (1993), and Lee and Hwang (2003): strategic wasteful expenditures. Strategic wasteful expenditure is self-interested expenditure that provides fringe benefits to the firm, such as goldplating, excessive advertising, and marketing, and excessive employee compensation. That is, even though \( C(Q) \) is the minimum amount necessary to produce output and thus \( C(Q) \) would constitute the firm’s total expenditure in the absence of taxation, the firm may choose to make self-interested expenditure, willfully engaging in waste and abuse. In order to incorporate this possibility, let \( e \) \((\geq 0)\) denote strategic expenditure by the firm which is not directly related to production activities. Thus, total expenditure may exceed minimal production cost by strategic expenditure: \( E = C(Q) + e \). Let the firm’s self-interested benefits from \( e \) be \( B(e) \) when the firm engages in strategic expenditure, where \( B(0) = 0 \), \( 0 < B' < 1 \), and \( B'' < 0 \). Note that the tax authorities can observe only total expenditure \( E \) but cannot tell production costs and strategic expenditure apart.

The monopolist might also make a decision about tax avoidance under uncertainty. The firm can evade profit tax liability by expenditure overstatement \( \delta \) \((\geq 0)\), which is either detected with probability \( \beta \) or remains undiscovered with probability \( 1 - \beta \). Let the firm’s after-tax profit when the tax authority does not detect the expenditure overstatement be \( \Pi^u \) and the after-tax and after-penalty profit when the authority successfully detects the evasion be \( \Pi^p \). Detection of under-reported profits involves a penalty \( f \) \((> 1)\), which is applied by the tax authority to the unreported portion of actual profit.

As formulated by Wang and Conant, assuming that both the probability of detection \( \beta \) and the penalty rate \( f \) are exogenously fixed, the firm’s net profit when it overstates its expenditure and is not detected by the authorities is:

\[
\Pi^u(Q,e,\delta) = R(Q) - E - t[R(Q) - (1 + \delta)E] = (1 - t)[R(Q) - C(Q) - e] + t\delta[C(Q) + e],
\]

where \( R(Q) = P(Q)Q \) and \( \delta E \) is overstated expenditure.

Conversely, if the firm’s expenditure overstatement \( \delta E \) is discovered by the tax authorities, the monopolist pays the penalty for the unpaid tax, \( t\delta E \), and thus its net profit is:

\[
\Pi^p(Q,e,\delta) = \Pi^u(Q,e,\delta) - f t\delta E.
\]
Incorporating the possibility of strategic expenditure leading to benefits, the objective of the firm is the sum of expected profit and these benefits, which can be written as follows:

$$\Pi(Q,e,\delta) = (1-\beta)\Pi^+(Q,e,\delta) + \beta \Pi^+(Q,e,\delta) + B(e)$$

$$= \Pi^+(Q,e,\delta) - ft\delta E + B(e)$$

$$= R(Q) - E - t(R(Q) - (1+\delta)E + f\delta E) + B(e).$$

Notice that the monopolist is assumed to be risk-neutral, as addressed by Kreutzer and Lee (1986, 1988) and Kim (1997), and thus it chooses its optimal output level $Q$, strategic expenditure level $e$, and expenditure overstatement factor $\delta$ so as to maximize the objective function in (1).

Assuming that the first-order necessary conditions for the output $Q$ and strategic expenditure $e$ have interior solutions and that the second-order sufficient conditions are also satisfied, the maximal conditions for solutions can be written as follows:

$$\Pi'_Q = (1-t)(R'(Q) - C'(Q)) + t\delta(1-\beta f)C'(Q) = 0,$$

$$\Pi'_e = B'-1-t + t\delta(1-\beta f) = 0,$$

where subscripts here and henceforth denote partial differentiation. Then, it is immediate from (2) that:

$$R'(Q) - C'(Q) = -t\delta(1-\beta f)C'(Q)/(1-t).$$

Notice that in the absence of a tax (i.e., when $t = 0$), the optimal output level will be found where $MR = MC$, yielding the familiar result that a profit tax is neutral. In addition, since $\Pi'_e = B'-1 < 0$, the strategic wasteful expenditure disappears, which is the standard result of cost minimization without profit taxation.

We now consider the equilibrium outcome in the tax incidence in (2) and (3). To do this, we need to consider the first-order condition for the tax evasion, $\Pi'_e = tE(1-\beta f)$, the sign of which depends on the value of $\beta f$.

First, if $\beta f = 1$, the optimal output level will be found where $MR = MC$, and tax evasion is indeterminate, i.e., $\Pi'_e = 0$ for all $\delta$. But the strategic wasteful expenditure can occur if $B' = 1-t$. This implies that even though the firm’s optimal production will be determined independent of the cost-statement decision, the firm may engage in strategic wasteful expenditure under profit taxation.

Second, if $\beta = 0$ or $f = 0$, the after-tax profit-maximizing output is greater than the output which would be produced in the absence of a tax ($MR < MC$) and tax evasion occurs, i.e., $\Pi'_e = tE > 0$, which implies that $\delta > 0$. Notice that this is the case of Kreutzer and Lee (1986), where it is perfectly legal for a firm to overstate its expenditure by some specified amount. As a result, the strategic expenditure will occur provided $B' = 1-t-t\delta$.

Finally, as long as $\beta f \neq 1$, the sign of $R'(Q) - C'(Q)$ is opposite to the sign
of $\delta(1-\beta f)$. With $\beta f < 1$, we have $\Pi_x = t(1-\beta f)E > 0$, which indicates that the optimal $\delta$ is positive. Thus, we obtain $MR < MC$, which implies that the after-tax profit-maximizing output $Q^*$ will be larger than the before-tax profit maximizing output $Q^-$. Notice that in this case, the optimal strategic expenditure $e'$ can be found to be:

$$1 - B'(e') = t(1 + \delta(1 - \beta f)).$$  \hspace{1cm} (5)

Conversely, if $\beta f > 1$, we have $\Pi_x = t(1-\beta f)E < 0$, indicating that the optimal $\delta$ must be zero. This implies that $MR = MC$, which in turn implies that the production decision and the evasion decision are, in this case, separable: $Q'' < Q^-$. And the optimal strategic expenditure $e''$ can be found as follows:

$$1 - B'(e'') = t.$$  \hspace{1cm} (6)

Comparing (5) and (6), since $B^* < 0$, the level of strategic expenditure under $\delta = 0$ is smaller than that under the formal case where the optimal $\delta$ is nonzero: $e'' < e'$. This represents a trade-off between production efficiency and cost efficiency when strategic behaviors are considered.

3. Extension and Discussion

In this section, we extend the analysis of the basic model to the case where the probability of detection $\beta$ and the penalty tax rate $f$ are functions of the expenditure over-reporting factor $\delta$, i.e., $\beta = \beta(\delta)$ and $f = f(\delta)$, where $\beta' > 0$, $\beta'' > 0$, $f' > 0$, and $f'' > 0$. Here, we assume $\beta f < 1$ for the interior solutions for $\delta$ in the extended model.

For analytic concreteness, we borrow the common assumption in a standard principle-agent model where the tax authority can announce and commit to its audit rule before the firm reports its expenditures. We also assume that audit costs are negligible and that the budget for the audit does not constrain the tax authorities. This implies that we don’t approach the game-theoretic equilibrium where the tax authorities cannot commit to its audit rule.

In this case, including the first-order conditions for the output in (2) and for strategic expenditure in (3), we can define the optimal level of tax evasion:

$$\Pi_x = tE(l - \beta f - f'\delta\beta - f\delta\beta') = 0.$$  \hspace{1cm} (7)

This yields the optimal expenditure overstatement factor:

$$\delta = \frac{1 - \beta f}{f'\beta + f\beta''}. \hspace{1cm} (8)$$

Notice that if $\beta f = 1$, the optimal output level is independent of tax evasion, which is indeterminate while the strategic wasteful expenditure occurs at the
optimum. However, if $\beta f < 1$, there might be a trade-off between production efficiency and cost efficiency under profit taxation. Specifically, substituting $\hat{\delta}$ in (8) into (2) and (3), we obtain:

$$R'(\hat{\delta}) - C'(\hat{\delta}) = -\frac{t \hat{\delta}}{(1-t)}(f^* \beta + \beta f'),$$

(9)

$$1 - B'(\hat{\epsilon}) = t(1 + \hat{\delta}(f^* \beta + \beta f')).$$

(10)

Therefore, only if $f^* \beta + \beta f' = 0$ are the production and evasion decisions separable, but the strategic expenditure may occur under profit taxation, i.e., $R'(Q) = C'(Q)$ and $B'(\hat{\epsilon}) = 1 - t$. Yaniv (1996) considered the case where the probability of detection and the penalty tax rate are associated with the amount of cost overstatement, where $\beta(t^*E)$ and $f(\hat{\delta}E)$. Then, from the first-order condition, we can show that $R'(Q) = C'(Q)$ holds, i.e., a profit tax is neutral to the optimal output. However, the level of strategic expenditure depends on the benefit derived from waste as well as the profit tax rate. Only if the marginal benefits of strategic expenditure or the profit tax rate is sufficiently low will the strategic expenditure be absent. In all other cases, the strategic expenditure appears.

On the other hand, if $f^* \beta + \beta f' > 0$ the after-tax profit-maximizing output will be larger than the output that would maximize profits in the absence of a tax. This implies that it is possible to employ profit taxation as an effective policy instrument to expand output levels. However, the strategic expenditure is determined by the conditions $f^* \beta + \beta f' = 0$ and $1 - B' = t(1 + \hat{\delta}(f^* \beta + \beta f'))$, which would not occur in the absence of a tax.

For comparative static analysis, we have the following second-order sufficient conditions, which should be negative definite:

$$H = \begin{bmatrix}
\Pi_{QQ} & \Pi_{Qe} & \Pi_{Q\delta} \\
\Pi_{eQ} & \Pi_{ee} & \Pi_{e\delta} \\
\Pi_{\delta Q} & \Pi_{\delta e} & \Pi_{\delta\delta}
\end{bmatrix},$$

where

$$\Pi_{QQ} = (1 - t)(R^* - C^*) + t \hat{\delta}(1 - \beta f)C^* < 0,$$

$$\Pi_{ee} = B^* < 0,$$

$$\Pi_{\delta\delta} = -t(2 \beta f' + 2 \beta f' + 2 \beta f' \hat{\delta} + \beta f' \hat{\delta} + \beta^* f' \hat{\delta})E < 0,$$

$$\Pi_{Q\delta} = \Pi_{Qe} = \Pi_{eQ} = \Pi_{e\delta} = \Pi_{Q\delta} = \Pi_{\delta e} = 0.$$

Using the implicit function theorem, if we hold all the exogenous variables and parameters fixed except for $t$, we obtain the following system equations:
The solutions are found by Cramer’s rule to be:

\[
\frac{\partial Q}{\partial t} = \frac{-\Pi_{\omega} \cdot \Pi_{e} \cdot \Pi_{\omega}}{[H]} > 0,
\]

\[
\frac{\partial e}{\partial t} = \frac{-\Pi_{\omega} \cdot \Pi_{e} \cdot \Pi_{\omega}}{[H]} > 0,
\]

\[
\frac{\partial \delta}{\partial t} = 0.
\]

Therefore, higher profit tax rates correspond to higher output levels and higher strategic expenditures, but the fraction of over-reporting remains unchanged. This implies that under profit taxation, the opposite tax effects on efficiency should be taken into consideration. The larger \( t \), the greater the production efficiency from expanding output beyond a no-tax level, but the greater the cost inefficiency from expanding strategic expenditure: there is a trade-off between production efficiency and cost efficiency.

Finally, from a normative perspective, we now seek an appropriate profit tax level and discuss some properties of optimal taxation. We assume that the tax authority decides the profit tax level to allocate tax resources in order to maximize social welfare. The use of this normative criterion would be consistent with the general literature on optimal taxation and would appear to be natural in the context of a model where the tax authority can commit to its policy before the firm reports its expenditures.

Then, the utilitarian tax authority desires to find an optimal profit tax rate to maximize the following expected social welfare function:

\[
W(Q,e,\delta,t) = \Pi(Q,e,\delta,t) + V(Q) + (1 - \alpha)G(Q,e,\delta,t),
\]

where \( \alpha \ (0 < \alpha < 1) \) is the tax distortion rate as an opportunity cost for the public funds, \( \Pi(Q,e,\delta,t) \) is the sum of monopolist’s expected profit and benefits in (1), \( V(Q) \) is consumer’s surplus, where
\[ V(Q) = \int_0^Q P(u) du - R(Q), \]

and \[ G(Q,e,\delta,t) \] is the government’s tax revenue, where

\[ G(Q,e,\delta,t) = t(R(Q) - (1 + \delta)E + \beta f\delta E). \] (15)

Some discussion of properties of the tax revenue in (15) is in order. First, \[ \partial G/\partial Q = t(R' - (1 + \delta)C' + \delta)C' + \beta fC' = (R' - C')/t < 0. \] Thus, the chain effect of the tax rate on tax revenue through output is negative, i.e., \( (\partial G/\partial Q) \cdot (\partial Q/\partial t) < 0. \)

Second, \[ \partial G/\partial e = -t(1 + \delta(1 - \beta f)) < 0. \] Thus, the chain effect of the tax rate on tax revenue through wasteful expenditure is negative, i.e., \( (\partial G/\partial e) \cdot (\partial e/\partial t) < 0. \)

Finally, \[ \partial G/\partial \delta = -tE(1 - \beta f) < 0 \] and \[ \partial G/\partial t = (R - (1 + \delta(1 - \beta f))E) > 0. \]

The authorities will have to take into account the tax effect on the monopolist’s responses in choosing the optimal tax, so that the optimal tax maximizes the welfare function as follows:

\[ W(Q(t),e(t),\delta(t);t) = \int_0^{Q(t)} P(u) du + B(e(t)) - C(Q(t)) + e(t) - \alpha G(Q(t),e(t),\delta(t),t). \] (16)

Assuming interior solutions, the optimal tax must now satisfy:

\[ \frac{dW}{dt} = \frac{\partial W}{\partial Q} \frac{\partial Q}{\partial t} + \frac{\partial W}{\partial e} \frac{\partial e}{\partial t} + \frac{\partial W}{\partial \delta} \frac{\partial \delta}{\partial t} + \frac{\partial W}{\partial t} = 0. \] (17)

This optimality condition can be rewritten as follows:

\[ (P - C') \frac{\partial Q}{\partial t} + (B' - 1) \frac{\partial e}{\partial t} = \alpha \left( \frac{\partial G}{\partial Q} + \frac{\partial G}{\partial e} + \frac{\partial G}{\partial \delta} + \frac{\partial G}{\partial t} \right). \] (18)

A few remarks are in order. First, if \( \beta f = 1 \), we have \( \delta = 0 \) from (8) and the output decision of the monopolist is separable from the evasion decision. Therefore, a profit tax is neutral to the output level (i.e., \( \partial Q/\partial t = 0 \)), and thus the optimal tax rate balances its effects on wasteful expenditure and government tax revenue:

\[ t^* = \frac{(1 - B') \partial e/\partial t + \alpha \partial G/\partial t}{\alpha(1 + \delta(1 - \beta f)) \partial e/\partial t}, \]

where \( \partial G/\partial t \) and \( \partial e/\partial t \) are positive.

Second, if \( \beta f < 1 \) and \( \alpha = 0 \), the optimal tax rate balances its effects on output and wasteful expenditure since tax revenue is neutral to society:

\[ t^* = \frac{(P - C') \partial Q/\partial t}{(1 + \delta(1 - \beta f)) \partial e/\partial t}, \]
where \( \partial Q/\partial t \) is positive. In particular, for the case of linear market demand and constant marginal cost, we obtain the following optimal tax formula:

\[
t'(1-t') = \frac{B'(P-C')(R'-C')}{R'[1+ \delta(1-\beta f)]}.
\] (19)

Note that the RHS of equation (19) is positive, which implies that the optimal tax \( t' \in (0,1) \) and there might be more than two optimal taxes.

Finally, for a sufficiently small size of \( \alpha \), even though there might be more than two optimal tax rates, we conjecture that the smaller tax rate is better for society when we take the negative tax revenue effect in (16) into consideration. Furthermore, if output distortion is severe (i.e., \( P-C' \) is large in (19)), the optimal tax rate should be large to increase production efficiency. On the other hand, if wasteful expenditure is so large that tax avoidance is severe (i.e., \( -B' \) is small in (19)), the optimal tax rate should be small to lessen cost inefficiency.

4. Concluding Remarks

The study on tax evasion is a persistent area of debate in the public economics, and characterizing and explaining the patterns of tax avoidance are obviously important to tax authorities. However, tax evasion is difficult to measure because firms often undertake substantial efforts to conceal their evasion.

This paper reexamines the model of tax evasion and output decisions of a monopolist under the realistic situation where the potential for moral hazard of the strategic firm behavior exists. We characterize strategic firm behavior under profit taxation and investigate the firm’s wasteful expenditure for tax avoidance. We show that when the probability of the under-reporting profits being detected is a function of the cost-statement factor, the monopolist’s decisions on output and expenditure are no longer separable from the evasion decision. We also find that there is a trade-off between production efficiency and cost efficiency in the context of profit taxation. Finally, we derive the optimal profit tax rate and analyze several properties of optimal profit taxation.

For future research, empirical issues on tax compliance with behavioral hypotheses and policy implications should be thoroughly investigated. In particular, a broadening of the empirical database will improve the power of statistical tests of theoretical models. As a different approach to modeling tax evasion of the firm, the game-theoretic model that is based on the interaction of strategies will stimulate the policy questions of tax avoidance. (On this point, see the survey paper of Andreoni et al. (1998).)

References


