Performance Measurement Technologies and the Trade-off of Risk and Incentives

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Abstract
This paper analyzes a principal’s decision of when to invest in a performance measurement technology. Interestingly, higher risk may make such an investment less likely. In this case, the strength of incentives certainly decreases. If, however, an increase in risk induces an initially non-investing principal to invest, incentive strength may increase.

Key words: complements; risk; incentive strength; performance measurement system

JEL classification: D81; J33; M52

1. Introduction

Although agency theory predicts a negative relation between the strength of incentives and risk affecting the outcome of a task, Prendergast (2002) documents that empirical studies rather find a positive one. He subdivides the corresponding empirical literature into the following four categories: executives, sharecroppers, franchisees, and others. While empirical studies for the first and last category are partly inconsistent, empirical studies for the second and third category clearly indicate a positive relationship between risk and incentives.

Prendergast (2000, 2002) offers some explanations for the ambiguity of theoretical and empirical findings. Most importantly, he argues that a firm’s decision of whether or not to delegate responsibility to an agent depends on how risky or uncertain (we use these terms as synonyms) the setting the agent works in is. Assuming that the agent is in possession of more accurate information concerning the setting, the firm will be likely to assign a task to the agent and to monitor her inputs if there is little uncertainty. In more uncertain settings, on the other hand, the firm is likely to delegate responsibility to the agent in order to make use of her superior information. Then, an output-based compensation scheme has to be

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installed to provide the agent with appropriate incentives.

A similar explanation is given by Baker and Jorgensen (2003). They distinguish between two types of uncertainty, which they label noise and volatility. Noise denotes uncertainty that does not affect the agent’s optimal action, while volatility denotes uncertainty that does so. Technically, the authors denote uncertainty as volatility if two conditions are fulfilled. First, the error term is multiplied with the agent’s action, i.e., it has an effect on her marginal product. Second, uncertainty is resolved to the agent after she signs a contract with the principal but before she chooses her action. With this distinction, risk can increase for two reasons: an increase in noise or in volatility. In the former case, incentive strength always decreases. In the latter case, incentive strength may increase. The intuition for this result is as follows. If the exact degree of volatility is known to the agent before she chooses her action, she will possess valuable information about the setting that the principal does not have. Then, the principal has to give the agent incentives to use this information in a sensible way. If the variance of the volatility term increases, the information advantage of the agent becomes more significant and the principal may choose higher incentives.

Note that both explanations have one important feature in common. In both papers, the agent is assumed to possess information that the principal does not have. The value of this information is positively related to the degree of uncertainty of the production process. As a consequence, the principal may propose a high-powered incentive scheme if risk is high in order to ensure efficient information use.

In this paper, we give a different explanation for the empirical findings. We assume that the principal might spend resources to increase the accuracy of the agent’s measured performance; that is, she could invest in a better monitoring technology. Examples of such monitoring technologies are ubiquitous: Lazear (2000) reports on Safelite Glass Corporation, a successful installer of automobile glass, which uses a sophisticated computerized information system to keep track of employee performance. Similarly, the balanced scorecard, as originated by Kaplan and Norton (1992), helps principals to better identify agent performance by establishing several criteria the agents have to meet. A very precise balanced scorecard performance measurement system may be able to mitigate inference problems that arise when only imperfect measures (such as output) of the agent’s performance exist. Note in this context the importance of the balanced scorecard outcome for an employee’s compensation. As Kaplan and Norton (1996, 2000) observe, 13 of 15 firms making use of balanced scorecards tie their employees’ wages to this outcome.

The introduction of a possible investment in a performance measurement system leads to several new results. First, there is a complementarity between such an investment and the strength of incentives. In particular, we will see that it is more worthwhile to undertake the investment the higher the incentive strength. Similarly, the strength of incentives is higher in case the investment has been undertaken than if it has not. Building on this complementarity, the positive trade-off between risk and incentives can be explained: if an increase in risk induces an originally
non-investing principal to invest in a monitoring technology, there are two countervailing effects on the strength of incentives. First, there is the direct negative effect already highlighted in the literature. Second, there is the positive complementarity effect. If the latter effect dominates, incentive strength increases. This means that the model provides a possible explanation for the empirical findings described above.

A further implication of the model is that the principal-agent model typically used in the literature may be too simplistic in its assumptions and may overlook important factors affecting the compensation decision. The decision of whether to invest in a performance measurement technology is one such factor. Other factors may be decisions of whether to train the agents, of whether to regularly install new machines, or decisions concerning product marketing, e.g., if the agents belong to the firm’s sales force.

The remainder of the paper is organized as follows. The next section presents the model and the model results. Section 3 contains a discussion and Section 4 concludes.

2. The Model

2.1 Description of the Model and Notation

Consider a model with a risk-neutral principal and a risk-averse agent. The agent chooses effort $e$ at cost $C(e)$ and so contributes to firm value. This contribution to firm value accrues to the principal and is given by the function $y = f(e)$. In order to derive several closed-form solutions, we assume $f(e) = e$ and $C(e) = 0.5e^2$. Note, however, that these assumptions are not crucial for our results. Let effort and, accordingly, the agent’s contribution to firm value be unverifiable to third parties like a court (see Baker et al., 1994, for similar assumptions). The principal receives an objective and verifiable signal $s \in \{s_0, s_1\}$ of the agent’s performance. The accuracy of this signal depends on whether the principal has undertaken an investment in a performance measurement technology. This investment costs $k$ (measured in monetary terms) and changes the available signal from:

$$s_0 = e + \epsilon$$  \hspace{1cm} (1)

to the more accurate signal:

$$s_1 = e + \theta$$  \hspace{1cm} (2)

The variables $\epsilon$ and $\theta$ denote uncorrelated error terms, which are normally distributed with the same mean $\mu$. For simplicity and without loss of generality we assume $\mu = 0$. The variances of the two error terms differ. Let $\sigma^2$ denote the variance of $\epsilon$ and $\lambda \sigma^2$ the variance of $\theta$ with $\lambda < 1$. Here $\sigma$ serves as a
measure of uncertainty. The higher $\sigma$, the more difficult it is to deduce the agent’s effort from the signal observation. For notational convenience, define as $I$ an indicator variable equaling $\lambda$ if the principal chose to install the monitoring technology and 1 otherwise. Further, let the actually incurred investment costs be given by $k$. Hence, $k$ equals $k$ if the investment in the performance measurement system has been undertaken and 0 otherwise.

The principal offers the agent a wage contract consisting of a fixed wage and a variable component. This wage contract is given by:

$$w = \alpha_0 + \alpha_1 s,$$

where we refer to $\alpha_1$ as the piece rate. This piece rate measures incentive strength. The restriction on linear wage contracts can be justified by interpreting the current model as a reduced-form game of the dynamic model by Holmström and Milgrom (1987). As shown in their paper, linear wage contracts are optimal under the current model assumptions.

Finally, the utility function of the agent is assumed to be exponential with constant absolute risk aversion and equals:

$$U(\nu) = 1 - \exp(-\nu \gamma),$$

with $\gamma > 0$ as the Arrow-Pratt measure of constant absolute risk aversion and $\nu$ as the agent’s wealth. The latter is given by the wage payment minus costs entailed by effort. Further, the agent is assumed to have an outside option that gives him a reservation utility $U$, which is normalized to zero.

The timing of the model is as follows. At date 1, the principal decides whether or not to invest in the performance measurement system. This decision is observable by the agent. At date 2, the principal offers the agent a wage contract, which the agent accepts or rejects at date 3. If she rejects the contract, the game will end. Otherwise, the agent will choose her effort at date 4. At date 5, nature chooses a realization of the error term, and at date 6 payments are made.

### 2.2 Solution to the Model

The model is solved by backward induction. Assuming that the agent has accepted the contract, she chooses her effort such that her certainty-equivalent $CE = \alpha_0 + \alpha_1 e - 0.5 e^2 - 0.5 (\alpha_1)^2 \gamma I_e \sigma^2$ is maximized. Optimal effort thus equals $\alpha_1$. The principal then determines optimally the piece rate $\alpha_1$, while $\alpha_0$ is chosen such that the agent’s participation constraint binds. Hence, the principal maximizes overall welfare, which is given by the agent’s contribution to firm value minus effort costs, the risk premium, and investment costs. Her (simplified) maximization problem is:

$$\max_{\alpha_0} E[\sigma] = \alpha_1 (1 - 0.5 \alpha_1) - 0.5 (\alpha_1)^2 \gamma I_e \sigma^2 - k.$$  

(5)
Note first that the investment in the performance measurement technology is complementary to incentive strength: it becomes more worthwhile the larger $\alpha$. Similarly, the piece rate will be higher, ceteris paribus, if the principal installs a monitoring technology than if she does not.

The solution to the principal’s maximization problem is derived by computing the first-order condition, which is equivalent to:

$$\alpha = \frac{1}{1 + \gamma I_i \sigma^2}. \quad (6)$$

The second-order condition is satisfied. Combining (5) and (6), we obtain an expression for profit, which only depends on the principal’s investment decision. This expression is given by:

$$E[\pi] = \frac{0.5}{1 + \gamma I_i \sigma^2} - k. \quad (7)$$

From (7), it is easy to see that the principal will choose to invest in the performance measurement technology if the following condition holds:

$$K \leq 0.5 \frac{\gamma \sigma^2 \gamma (1 - \lambda)}{(1 + \gamma \lambda \sigma^2)(1 + \gamma \sigma^2)} \approx Y. \quad (8)$$

Lemma 1 offers some interesting comparative statics.

**Lemma 1:** $\frac{\partial Y}{\partial \lambda} < 0$, $\frac{\partial Y}{\partial \sigma} > 0 \Leftrightarrow 1 + \gamma^2 \lambda \sigma^4$, $\frac{\partial Y}{\partial \gamma} > 0 \Leftrightarrow 1 + \gamma^2 \lambda \sigma^4$.

**Proof:** Consider the derivative of $Y$ with respect to $\lambda$. This derivative equals $\frac{\partial Y}{\partial \lambda} = 0.5[-\gamma \sigma^2((1 + \gamma \lambda \sigma^2)(1 + \gamma \sigma^2) - \gamma \sigma^2(1 - \lambda)(1 + \gamma \sigma^2)\gamma \sigma^2)]/(1 + \gamma \lambda \sigma^2)(1 + \gamma \sigma^2)$.

The numerator is strictly negative and so is $\frac{\partial Y}{\partial \lambda}$. Furthermore, we also have that the derivative of $Y$ with respect to $\sigma$ is given by $\frac{\partial Y}{\partial \sigma} = \frac{0.5[\gamma \sigma^2((1 + \gamma \lambda \sigma^2)(1 + \gamma \sigma^2) - \gamma \sigma^2(1 - \lambda)(1 + \gamma \lambda \sigma^2)\gamma \sigma^2)]/(1 + \gamma \lambda \sigma^2)(1 + \gamma \sigma^2)}$. Rearranging the condition that $\frac{\partial Y}{\partial \sigma} > 0$ we can obtain that $2(1 + \gamma \lambda \sigma^2)(1 + \gamma \sigma^2) - (1 + \gamma \lambda \sigma^2)2 \sigma^2 - (1 + \gamma \sigma^2)2 \lambda \sigma^2 > 0$, which is equivalent to $1 > \gamma^2 \lambda \sigma^4$. Finally, consider the derivative with respect to $\gamma$. We find $\frac{\partial Y}{\partial \gamma} = 0.5[\gamma \sigma^2((1 + \gamma \lambda \sigma^2)(1 + \gamma \sigma^2) - \gamma \sigma^2(1 - \lambda)(1 + \gamma \lambda \sigma^2)\sigma^2 + (1 + \gamma \sigma^2)\lambda \sigma^2)]/(1 + \gamma \lambda \sigma^2)$. It will be positive if $(1 + \gamma \lambda \sigma^2)(1 + \gamma \sigma^2) - \gamma((1 + \gamma \lambda \sigma^2)\sigma^2 + (1 + \gamma \sigma^2)\lambda \sigma^2) > 0$, which can be rewritten as $1 > \gamma^2 \lambda \sigma^4$.

Lemma 1 has several implications. First, it implies that the principal is more likely to invest in the measurement technology the lower $\lambda$. This is very intuitive. The lower $\lambda$ the more the investment filters noise and the higher the direct gain from the investment. More interesting is the result that the investment may become less likely the higher $\sigma$ or $\gamma$. An increase in $\sigma$ or $\gamma$ affects the principal’s
investment decision in two ways. On the one hand, there is a direct effect. If $\sigma$ or $\gamma$ is higher, the investment’s absolute effect on the risk premium is higher and the investment becomes more worthwhile. This can be seen, for a fixed value of $\alpha$, from the profit formula in (5). On the other hand, there is an indirect effect as well, namely the complementarity effect. An increase in $\sigma$ or $\gamma$ affects the optimal piece rate, which, due to the complementarity between the investment and $\alpha$, in turn affects the investment decision. If $\frac{\partial \alpha}{\partial \sigma} > 0$ (or $\frac{\partial \alpha}{\partial \gamma} > 0$), both effects work in the same direction and the investment becomes more likely. If, however, the opposite relation holds, the two effects are countervailing and the principal may become less likely to undertake the investment.

From Lemma 1, the following proposition can be derived.

**Proposition 1**: An increase in risk may lead to higher or lower $\alpha$.

**Proof**: Consider the following two cases separately. In the first case, the principal initially chose to invest in the measurement technology. Let $\sigma$ increase, say from $\sigma_1$ to $\sigma_2 > \sigma_1$. Then, Lemma 1 states that two things may happen. First, the principal may still undertake the investment. It follows that $\alpha_1$ decreases from $\alpha_{i1} = 1/1 + \gamma \sigma_1^2$ to $\alpha_{i2} = 1/1 + \gamma \sigma_2^2$. Second, the principal may decide to abandon the investment. In this case, the piece rate would also decrease, namely from $\alpha_{i1} = 1/1 + \gamma \sigma_1^2$ to $\alpha_{i2} = 1/1 + \gamma \sigma_2^2$. In the second case, the principal initially chose not to invest in the measurement technology. Again, two situations may arise. The principal may still choose to forego the investment opportunity. Then, the piece rate would decrease from $\alpha_{i1} = 1/1 + \gamma \sigma_1^2$ to $\alpha_{i2} = 1/1 + \gamma \sigma_2^2$. However, she may also be induced to invest in the monitoring technology. Here, incentive strength will change from $\alpha_{i1} = 1/1 + \gamma \sigma_1^2$ to $\alpha_{i2} = 1/1 + \gamma \sigma_2^2$. Hence, $\alpha_{i2}$ is bigger (smaller) than $\alpha_{i1}$, if $\lambda \sigma_2^2 < (>) \sigma_1^2$. It remains to be shown that each of the cases considered in this proof indeed may occur for some parameter constellations. We demonstrate this only for the case where incentive strength increases after the increase in risk, as this is the most interesting case. It is easy to find examples for the remaining cases. Suppose that $\gamma = \sigma_1 = 1$, $\lambda = 0.25$, $\tilde{k} = 0.16$, and $\sigma_2 = 1.5$. Here, $Y_i = 0.15$ so that, initially, the principal did not undertake the investment. Further, $Y_i = 0.166$. Hence, the increase in risk induces the principal to undertake the investment. Finally, $0.5625 = \lambda \sigma_1^2 < \sigma_1^2 = 1$ implying that, after the increase in risk, incentive strength is higher.

The intuition behind Proposition 1 is as follows. An increase in risk affects the piece rate in two ways. First, there is the traditional insurance effect. If risk increases, it will become more costly to incentivize the risk-averse agent so that incentive strength decreases. However, the piece rate will be set higher if the principal decides to invest in a performance measurement technology, as both instruments are complementary. Hence, if an increase in risk induces the principal to install a monitoring technology, there will be two countervailing effects on the piece rate. If the complementarity effect dominates, the optimal piece rate will increase.
3. Discussion

The aim of this section is twofold. First, we show that the model results continue to hold if the investment decision is continuous rather than discrete. Then we discuss potential applications of the model and existing empirical evidence.

3.1 Continuous Investment

In this subsection, the investment in the performance measurement technology is assumed to be continuous. In particular, let the principal be able to select \( \lambda \in [0, 1] \) at a cost \( K(\lambda) \) with \( K(\lambda) \) decreasing in \( \lambda \) and satisfying \( K(1) = 0 \). The principal’s expected profit then changes from the one in (7) to:

\[
E[\pi] = \frac{0.5}{1 + \gamma \lambda \sigma^2} - K(\lambda) .
\] (9)

The principal chooses \( \lambda \) so as to maximize this profit, which leads to the first-order condition:

\[
-0.5\gamma \sigma^2 \frac{1}{(1 + \gamma \lambda \sigma^2)^2} - K'(\lambda) = 0
\] (10)

Note that the second-order condition is \( \gamma^2 \sigma^4 / (1 + \gamma \lambda \sigma^2)^3 < K'(\lambda) \) and requires \( K'(\lambda) \) to be sufficiently large. In what follows, this is assumed to be the case so that the solution to (10) indeed describes a maximum.

Unfortunately, it is impossible to obtain explicit solutions for \( \lambda \) even if we assume specific functional forms for \( K(\lambda) \). Further, applying the method of implicit differentiation to conditions (10) and \( \alpha_i = 1/(1 + \gamma \lambda \sigma^2) \) does not work here either, as this method yields comparative statics results that depend on endogenous variables. Nevertheless, we can show that Proposition 1 continues to hold in this alternative setting. To see this, let \( K(\lambda) = (1 - \lambda)^2 / 8 \) and \( \gamma = 1 \). Moreover, let \( \sigma \) increase from \( \sigma_1 = 0.5 \) over \( \sigma_2 = 0.6 \) to \( \sigma_3 = 0.7 \). In the first case, the optimal solution is given by \( \alpha_{i1} = 0.865 \) and \( \lambda_1 = 0.626 \). Similarly, the solutions for the second and third case are \( \alpha_{i2} = 0.854 \), \( \lambda_2 = 0.475 \), \( \alpha_{i3} = 0.932 \), and \( \lambda_3 = 0.148 \) (notice that the second-order condition is always satisfied in these examples). Hence, an increase in \( \sigma \) may still lead to a lower or a higher piece rate. This implies that the result in Proposition 1 does not depend on the assumption of a discrete investment possibility.

3.2 Application and Empirical Evidence

According to the model, higher risk may induce the principal to invest (more strongly) in a performance measurement technology and this may in turn lead to higher incentives. In principle, the model may therefore be applied to all situations,
where performance measures of different accuracy are used. For instance, CEOs are oftentimes compensated according to signals that adjust a CEO’s own firm’s performance by incorporating performance data of comparable firms from the same industry. The aim of this adjustment is to reward (or punish) a CEO for her true performance and not for common (systematic or random) shocks affecting the performance of firms in the whole industry. This adjustment reduces possible noise and so may help to save on risk premiums. On the other hand, the gathering of performance data of other firms is certainly costly so that there is a clear trade-off of the gains and costs of having a more accurate performance measure. Similar arguments can be made for franchising, where most franchisors collect lots of information from the franchisees to reduce potential noise, or for workers on lower hierarchy levels, whose performance is hard to measure, unless investments in performance measurement technologies are made. In general, the inclusion of information concerning the performance of others in similar positions helps to filter out common shocks, while methods focusing on individual performance (like performance evaluations of superiors or questionnaires sent to clients) may be better suited in filtering out idiosyncratic shocks.

Unfortunately, there exists no empirical study (to our knowledge) that analyzes how investments in performance measurement affect the trade-off between risk and the strength of incentives, though this is an interesting topic for future research. Nevertheless, there is some support for the model. First, the model predicts that higher incentives should come along with a better performance measurement technology. A nice confirmation of this prediction can be found in the recent discussion of the implementation of pay-for-performance systems in US health care. As stated in an issue brief by the Alliance of Community Health Plans (2005), health care costs in the US have been relentlessly rising, while at the same time the quality of health care has been extremely variable. To mitigate these problems, attention has recently focused on pay-for-performance systems, which should reward physicians and hospitals for good performance. In the discussion, it becomes quite clear that the main problem of these systems is the identification and development of appropriate performance measures. For example, Gregg et al. (2006) argue that hospitals in rural areas first need to set up a sophisticated performance measurement system before a pay-for-performance system could be implemented.

Finally, in the model, a necessary condition for the positive trade-off of risk and incentives is a positive relation between risk and investments increasing the accuracy of the agent’s measured performance. Some empirical support for this relation is given by Poppo and Zenger (2002). In a survey study, they find that firms having difficulties in measuring employee performance react by developing more complex contracts. These complex contracts could be understood as a means enabling the firms an accurate measuring and rewarding of employee performance.

4. Conclusion

In this paper, the principal-agent model is enriched by allowing the principal to
invest in a performance measurement system. In this way, the positive trade-off between risk and incentives can be explained. An increase in risk may induce the principal to install a monitoring technology, and this may in turn lead to higher incentives.

It would be interesting to introduce such an investment possibility into other models as well. For instance, Itoh (1994, 2001) discusses, in a similarly structured model, the benefits and costs of different forms of job design. He finds that a principal is unlikely to delegate a task in a high risk setting, as providing an agent with incentives is then very costly. Having the results of this paper in mind, we see that this prediction has to be treated with caution. In a high-risk setting, the principal may prefer to invest in a monitoring technology and then to delegate the task to handling the task herself.

Finally, investments in a monitoring technology might play an important role if a principal arranged a rank-order tournament between agents with limited liability analyzed in Kräkel (2006). In this case, the principal can induce high efforts either by increasing monetary rewards (the winner prize) or by reducing the impact of random factors. Consequently, accurate performance measurement systems might not only serve to ensure agents against income risks but also to incentivize the agents.

References


