Offshore Bidding and Currency Futures

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Abstract
In an interactive model of offshore bidding, two firms located in two different countries bid on a project in a third country under exchange rate uncertainty. Every firm benefits and provides a higher bid when both firms have hedging opportunities. Even if only one bidder has the hedging opportunity, both bidders gain through an increase in their expected utilities.

Key words: exchange rate; futures markets; uncertainty; game theory; multinational enterprise

JEL classification: F3; C7; D8

1. Introduction
The theory of foreign trade and investment has argued that exchange rate volatility is a significant factor in multinational decisions regarding foreign direct investment (FDI). However, empirical studies have found no significant correlation between exchange rate volatility and FDI (Crabb, 2002; Gorg and Wakelin, 2002). The primary reason stated in the literature is that foreign exchange risk can be offset by appropriate positions in currency forward or futures markets when such markets are available. The issue of foreign exchange risk is also present in more general settings of international transactions involving goods or financial assets. Empirical studies in such settings also indicate a relative lack of correlation between such foreign transactions and exchange rate risk with the same justification pointing to currency hedging in the forward or futures markets (Wei, 1999; Abanomey and Mathur, 2001; Allayannis et al., 2001; Haigh and Holt, 2002; Lioui and Poncet, 2002; Hagelin, 2003).
Despite the abundance of studies on the connection between foreign exchange rate risk and transactions involving foreign asset holdings, the literature on multinational bidding and its connection to foreign exchange volatility is rather limited. This can be partially attributed to the fact that bidding on a foreign project typically involves a game-theoretic process and is thus a fundamentally different exercise than an outright purchase of a foreign asset. Meanwhile, bidding on foreign development and construction projects is a growing area of multinational enterprise, in particular in industries such as oil and gas, chemical, and electric and nuclear power. Specialized multinationals now routinely bid on such projects in various countries. For example, in January 2008, Hythane Company, an Australian firm, outbid American and European competitors to win an international tender from Indian Oil Corporation to build the first hydrogen fuel station valued at $1 million. (http://www.fuelfcellworks.com/Suppage8235.html). Since the bidding round was announced in February 2008, 37 companies have expressed interest in the bid for Bangladesh offshore blocks. At least 9 of them were expected to submit their offers on May 7, 2008. (http://www.redorbit.com/news/137210/). In February 1998, five companies (from the US, Belgium, Singapore, Malaysia/New Zealand, and Bangladesh) participated in an international tender floated by Bangladesh Power Division’s Power Cell. Till now, the result has not yet been announced. (http://newsfrombangladesh.net/view.php). In addition, several companies are specialized in providing services and information for international tender opportunities, such as Tenders Zeal and TenderNews.com.

The existing literature on multinational bidding is focused primarily on the game-theoretic aspect of such biddings with particular applications of auction theory and overlooks the crucial role that currency futures markets play in such biddings (Moody, 1994; Porter, 1995). Kulatilaka and Marcus (1994) examined the exchange rate risk associated with international bidding and concluded that futures seem to dominate options as the primary vehicle of hedging exchange rate risk in such cases. Sercu and Uppal (1995, p. 180) and Lien and Wong (2004) suggest that options may play an important role in offshore bidding. Although we focus on futures and do not consider options explicitly in this study, intuitively the same arguments apply and the conclusions should remain qualitatively unchanged.

We study the implications that the presence of currency futures markets in some or all countries can generate for optimal bidding on foreign projects. We consider the case of two foreign firms located in two different countries that bid for a project in a third country. Ultimately, each bidding firm cares for profits in own currency. Under exchange rate uncertainty and certain symmetry conditions with state-dependent utilities, we employ a game-theoretic model to analyze the optimal bidding strategies of the two firms. We compare the optimal strategies that emerge from three possible settings: (i) no currency futures market exists in any of the two bidding firms’ countries, (ii) currency futures markets exit in both of the two bidding firms’ countries, and (iii) a currency futures market exists only in one of the two bidding firms’ countries. Several interesting results emerge. For instance, in case (ii), both firms engage in a partial hedge such that each firm’s futures position equals its
bid, adjusted by the probability of winning. Because a futures hedge reduces the risk associated with a bid, each firm submits a larger bid and draws a larger expected utility than those in case (i). In case (iii), the hedged firm always submit a larger bid and draws a larger expected utility than the unhedged firm. Moreover, a consequence of strategic interaction is demonstrated in case (iii) where both the hedged and unhedged firms increase their bids and improve their expected utility level relative to case (i), but they fare worse relative to the outcomes in case (ii). The three cases (i)-(iii) are discussed in Sections 2-4 below, respectively. In Section 5, we consider cross hedging scenarios. Concluding remarks appear in the last section.

2. The Benchmark Model

Suppose that two firms, located in countries A and B, bid competitively for a construction project in country C. Let $e_{t}$ denote the exchange rate between the currencies of countries A and C such that one dollar in country C can be exchanged for $e_{t}$ dollars in country A at time $t$. Similarly let $e_{t}$ denote the exchange rate between the currencies of countries B and C. Let $B_{k}$ denote the bid submitted by firm $k$ (denominated in country C currency) at time $0$, $k=1,2$. If successful, the firm will receive the bid amount at time 1. Otherwise, the firm has no gains and no losses. Let $u_{k}(\cdot)$ denote the utility function for firm $k$ in state $j$ where $j=1$ if the firm wins the bid and $j=0$ otherwise. Specifically, we allow the firm to evaluate its utility after the bidding outcome is announced (and allow different functional forms when the outcomes are different). Due to exchange rate fluctuations, both firms incur risk in uncertain revenues. When the firm loses the bid, we normalize the state dependent utility level to zero, i.e., $u_{k}(0)=0$. The expected utility for firm 1 is:

$$EU_{1} = Eu_{11}(B_{1}e_{11} - C_{1})P(B_{1},B_{2}),$$

(1)

where $C_{1}$ is firm 1’s production cost in its own currency, and $P(B_{1},B_{2})$ is the probability that firm 1 wins the bid when it bids $B_{1}$ and firm 2 bids $B_{2}$. That is, the firm’s expected utility is the utility from the profit of implementing the project discounted by the probability of winning the bid. Similarly, the expected utility for firm 2 is:

$$EU_{2} = Eu_{21}(B_{2}e_{21} - C_{2})[1 - P(B_{1},B_{2})],$$

(2)

where $C_{2}$ is firm 2’s production cost in its own currency.

We assume that each firm has a mean-variance expected utility function such that $Eu_{k}(W) = E(W) - (\lambda_{k} / 2)Var(W)$, where $\lambda_{k}$ is the risk aversion coefficient and $Var(\cdot)$ is the variance operator. Indeed, if $u_{k}(\cdot)$ adopts a quadratic functional form, then the expected utility approach reduces to the mean-variance method. A similar result applies when $u_{k}(\cdot)$ adopts an exponential functional form provided the underlying random variable is normally distributed. Alternatively, the mean-variance method can be treated as an approximation to the expected utility analysis.

Moreover, we adopt the following “contest success” function:
where \( P(B_1, B_2) \) is the probability that firm 1 wins the bid. If each firm submits the same bid, then each has a 50% chance of winning. If firm 1 bids higher, then its chance of winning is less than 50%. For example, if the bid from firm 1 is twice of that from firm 2, the chance that firm 1 wins is one-third. In addition, \( P(B_1, B_2) \) increases as \( B_1 \) decreases or as \( B_2 \) increases, i.e., a smaller bid or a larger rival bid increases the chance of winning. Under the above specification, the likelihood of winning for a firm who enters a larger bid is not zero. Note that Baye et al. (1993) discuss an alternative success function. Given (3), the expected utility for firm \( k \) is:

\[
EU_k = [B_k E(e_{k1}) - C_k - \theta_k B_k^*] [1 - B_k / (B_1 + B_2)],
\]

for \( k = 1, 2 \) where \( \theta_k = (\lambda_k / 2) \text{var}(e_{k1}) \). We consider the symmetric case such that \( \theta_1 = \theta_2 = \theta \), \( C_1 = C_2 = C \), and \( E(e_{k1}) = E(e_{k2}) \). Also, without loss of generality, let \( E(e_{k1}) = 1 \).

The solution to the above bidding game is characterized by a Nash equilibrium with both firms participating in the bidding. Let \((B_1^*, B_2^*)\) denote the bid pair. Participation requires that, under the stated bids, the participant will obtain higher expected utility level than the level obtained under nonparticipation. In addition, if firm 1 submits bid \( B_1^* \), firm 2 cannot do any better by submitting a bid different from \( B_1^* \). Similarly, if firm 2 submits bid \( B_2^* \), firm 1 cannot improve its expected utility by submitting a bid different from \( B_2^* \). In other word, no firm can unilaterally improve its expected utility by deviating from the equilibrium bid pair \((B_1^*, B_2^*)\).

Mathematically speaking, firm 1 chooses its bid to maximize the expected utility assuming firm 2’s bid, \( B_2^* \), is fixed. The resulting first-order condition is:

\[
B_1 - \theta B_1 (B_1 + 2B_2) + C = 0.
\]

Similarly, the first-order condition for firm 2’s maximization problem is:

\[
B_2 - \theta B_2 (2B_1 + B_2) + C = 0.
\]

Solving equations (5) and (6) simultaneously, we derive:

\[
B_1^* = B_2^* = \frac{1 + \sqrt{12\theta C + 1}}{6\theta},
\]

which determines a Nash equilibrium provided the participation constraint is satisfied. That is, the firm must achieve a higher expected utility level when participating in the bidding process than staying out. The expected utility from staying out is normalized to zero. Thus, for \((B_1^*, B_2^*)\) to be a Nash equilibrium, we require \( EU_k^* \), the value of \( EU_k \) when evaluated at \( B_1^* \) and \( B_2^* \), to be non-negative. After algebraic computations, we have:
for $k = 1, 2$. The participation constraint is therefore:

$$\theta C \leq 1/4.$$  \hfill (9)

A comparative static analysis shows that $\partial \mathcal{B}_k^0 / \partial C > 0$ and $\partial \mathcal{B}_k^0 / \partial \theta < 0$. A firm will raise its bid as costs increase and will lower its bid if the firm is more risk averse. In fact, given the success function, a risk neutral firm prefers to submit as large a bid as possible. Although a larger bid has a smaller chance of winning, the decrease in probability is not sufficient to offset the increase in revenue. While the firm prefers a large bid, aversion to exchange rate fluctuations prevents the firm from submitting a large bid as the risk is proportional to the square of the bid. Consequently, as the firm becomes more risk averse, a smaller bid is submitted. The increase in bid along with increasing cost is rather intuitive. Further, we have $\partial \mathcal{EU}_k^0 / \partial \theta < 0$ and $\partial \mathcal{EU}_k^0 / \partial C < 0$. That is, the firm is worse off at higher degrees of risk aversion or levels of cost.

Note that (5) and (6) have multiple solutions. First, we restrict ourselves to symmetric solutions as the firms are assumed to be symmetric. Second, the bid must be positive. These two criteria lead to (7) as the unique solution. Also note that the optimal bid is a function of the variance of the future exchange rate. However, it is independent of the realization of the future exchange rate. For each firm, the bid is submitted before the future exchange rate is realized. The probability of winning the bid is a function of the two submitted bids, and henceforth it is independent of the realization of the future exchange rate.

3. Currency Futures Markets

We now evaluate the case where a currency futures market is available in both countries A and B. At time 0, by selling a unit of the futures contract, firm 1 agrees to exchange a unit of country C currency for $f_0$ of its own currency at time 1. Let $x$ denote the units of futures contract firm 1 sells. The profit at time 1 is given by:

$$W_{01}^0 = (B - C) + (f_0 - e_1)x,$$

which is the operational gain plus the capital gain from futures trading if the firm wins the bid. Otherwise, the profit is:

$$W_{01} = (f_0 - e_1)x.$$  \hfill (11)

In other words, by selling currency futures contracts, firm 1 reduces its risk in case it wins the bid but incurs additional risk when it loses. The optimal futures position trades off the two effects. More specifically, firm 1 chooses $B$, and $x$ to maximize the expected utility function:
The first term is the expected utility when the firm wins the bid and the second is the expected utility when it loses the bid. The state-dependent utility assumption, particularly the assumption that the firm evaluates its utility after the bid outcome is announced, allows the expected utility to be decomposed into two components.

Similarly, by selling a unit of the futures contract at time 0, firm 2 agrees to exchange a unit of country C’s currency for $g$ of its own currency at time 1. Let $y$ denote the units of futures contract firm 2 sells. Then firm 2 will choose $B_2$ and $y$ to maximize the expected utility function:

$$EU_2 = Eu_2[B_2e_{12} - C_2 + y(g_0 - e_{12})][1 - P(B_1, B_2)] 
+ Eu_2[y(g_0 - e_{12})]P(B_1, B_2).$$

To proceed, we apply the mean-variance approach to both $u_{ak}$ and $u_{k1}$, $k = 1, 2,$ using the symmetry assumptions in the previous section. In addition, we assume that each futures market is unbiased: $E(e_1) = 0$ and $E(e_2) = 0$. That is, the currency futures rate is equal to the expected spot rate at the maturity date. Because we are concerned with the hedging function of the futures markets, these assumptions help remove the unnecessary complications from speculative trading by the firms. Consequently, in maximizing (12) over $x$ and $B_1$, the first-order conditions for firm 1 lead to:

$$x = B_1(B_1 + B_2),$$

$$B_2 - 3B_1 + B_1 B_2 + C = 0.$$  

By the symmetry assumptions, we also have:

$$y = B_2(B_1 + B_2),$$

$$B_1 - 3B_1 + B_1 B_2 + C = 0.$$  

Solving (15) and (17) simultaneously, we derive the optimal bid for either firm as:

$$B_1^* = B_2^* = \frac{1 + \sqrt{8OC + 1}}{4\theta}.$$  

The optimal futures position emerges by applying (18) in (14) and (16):

$$x^* = y^* = \frac{1 + \sqrt{8OC + 1}}{8\theta}.$$
Accordingly, at the Nash equilibrium the expected utility for each firm is:

\[ EU_k' = \frac{1}{2} \left\{ B_k' - C - \left( \theta / 2 \right) \theta (B_k')^2 \right\} = \frac{3 + 3\sqrt{8\theta C + 1}}{32\theta} - \frac{5C}{8}, \]  

(20)

for \( k = 1, 2 \). The participation restriction requires \( EU_k' \geq 0 \), that is:

\[ \theta C \leq 12/25. \]  

(21)

This restriction is easier to satisfy than that prescribed when futures markets are not available.

The properties of the optimal bid and the maximum expected utility are the same as those described in the previous section. Specifically, the bid increases with increasing cost or decreasing risk aversion whereas the maximum expected utility decreases when either factor increases. Moreover, by comparing (7) and (18), the effect of futures trading on the optimal bid is the same as that of a reduction in the risk aversion by a third. Upon comparing \( B_k' \) with \( B_k'' \), and \( EU_k' \) with \( EU_k'' \), Theorem 1 follows immediately.

**Theorem 1:** If for each bidder there is an unbiased currency futures market available for trading, then at the Nash equilibrium the firm’s bid increases and so does its expected utility when compared to the case where futures trading is not available.

When a firm submits a large bid, currency fluctuations become large should the firm win the bid. Risk aversion hence prevents the firm from submitting a large bid. Availability of futures trading allows the firm to hedge currency risk and consequently a larger bid is submitted. Note that, a larger bid increases the firm’s expected profit. Now that the risk is reduced and the return is increased, the firm achieves a higher expected utility.

**4. A Single Futures Market**

We now study the case where only one country, say country A, has a currency futures market. Then firm 1 has a hedge instrument but firm 2 does not. Firm 1’s objective function is given by (12) whereas firm 2’s is given by (4). The resulting first-order conditions are (14)-(15) and (6) respectively, which we solve for the Nash equilibrium. For clarity, we reproduce equations (14) and (6) below:

\[ B_2 - \theta B_1 \left( \frac{R_1 + 3B_2}{B_1 + B_2} \right) + C = 0, \]  

(22)

\[ B_1 - \theta B_2 (2B_1 + B_2) + C = 0. \]  

(23)

Let \( B_k'' \) denote the bid submitted by firm \( k \) at the Nash equilibrium, \( k = 1, 2 \). Theorem 2 describes the relationship between the two bids.
Theorem 2: At the Nash equilibrium $B_i^* > B_i'$. That is, the firm with hedging opportunities submits a larger bid than the firm without hedging opportunities.

Proof: From (22) and (23), we have:

$$B_i - \theta B_i (2B_i + B_j) = B_j - \theta B_j \left( \frac{B_i + 3B_j}{B_i + B_j} \right).$$ (24)

After algebraic manipulations, this leads to:

$$B_i' - B_j' = \theta (B_i^2 - B_i^3 - B_j^2 + 3B_iB_j^2).$$ (25)

Define $f(B_i) = B_i^2 - B_i^3 - \theta (B_i^2 - B_i^2 - B_j^2 + 3B_iB_j^2)$, a function of $B_i$ for a given $B_j$. Now, $f(B_i) < 0$ and:

$$f'(B_i) = 2B_i + 2\theta B_iB_j + 3\theta (B_i^2 - B_j^2) > 0,$$ (26)

whenever $B_i \geq B_j$. Also note that $f''(B_i) > 0$ for any $B_i$, $f(0) < 0$, and $f''(0) < 0$. Using these properties, we find $f(B_i)$ is U-shaped and intersects the $B_i$-axis at a point larger than $B_j$. As a result, $B_i^* > B_i'$

Intuitively, futures markets allow firm 1 to reduce its risk level (with an effect similar to that of a reduction in risk aversion coefficient). Firm 1, therefore, is more aggressive and provides a larger bid. Theorem 3 provides upper and lower bounds for the bids at Nash equilibrium.

Theorem 3: At the Nash equilibrium, $B_i^*>B_i^0=(1+\sqrt{12\theta C+1})/6\theta C$ and $B_i^*<B_i'=1+(\sqrt{8\theta C+1})/4\theta C$.

Proof: Define $g(B_i,B_j)$ and $h(B_i,B_j)$ to be the left-hand-sides of (22) and (23), respectively. We have $\partial h(B_i,B_j)/\partial B_j < 0$ and:

$$\frac{\partial h(B_i,B_j)}{\partial B_j} = 1 - 2\theta B_j \leq 1 - \frac{1+\sqrt{8\theta C+1}}{2} < 0,$$ (27)

when $B_j \geq B_j'$. Since $h(B_i,B_j) < 0$ when evaluated at $(B_i',B_j')$ at the Nash equilibrium, we must have $B_i' < B_i' = (1+\sqrt{8\theta C+1})/4\theta C$. Similarly, we can show both $\partial g(B_i,B_j)/\partial B_i > 0$ and $\partial g(B_i,B_j)/\partial B_j > 0$ when $B_i \leq B_i^0$. Using the property that $g(B_i',B_j') > 0$, at the Nash equilibrium, it must be the case that $B_i^* > B_i^0 = (1+\sqrt{12\theta C+1})/6\theta C$. In other words, the smaller bid cannot be larger than the level that prevails when both futures markets are available, and the larger bid must be larger than the level that prevails when neither futures market exists.

Intuitively, we would expect the following relations:

$$B_i^0 = B_i' \leq B_i^* \leq B_i^* = B_i'$$
where some of the stated relations have already been established analytically. The comparative static analysis is complicated although once again we expect that \( B_1 \) and \( B_2 \) to increase with decreasing risk aversion or increasing cost. Also, we expect firm 1 to achieve a higher expected utility level than firm 2. Because analytic solutions are not available in this case, we adopt a simulation method to examine the above conjectures. We consider three parameter configurations: \((\theta, C) = (0.05, 2), (0.05, 3), \) and \((0.1, 2)\). Note that we choose parameter values to satisfy the participation constraint in (10). Table 1 presents the Nash equilibrium outcomes for each of the three scenarios, no futures markets, two futures markets, and one futures market only in country A. In each case, we find that all of the above conjectures are verified. For example, a firm bids the most when there are two futures markets and the least when there is none. Similarly, the firm derives the highest expected utility in the former and the lowest in the latter. The equilibrium bid and hence the expected utility increase with increasing cost or decreasing risk aversion in each scenario.

<table>
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<tr>
<th>Outcomes</th>
<th>No Futures Markets</th>
<th>Two Futures Markets</th>
<th>One Futures Market</th>
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<tr>
<td><strong>Case 1: ( \theta = 0.05, C = 2 )</strong></td>
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<tr>
<td>Firm 1 bid</td>
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<td>1.5974</td>
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<td>Firm 1 bid</td>
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<td><strong>Case 3: ( \theta = 0.10, C = 2 )</strong></td>
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<td>Firm 2 expected utility</td>
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</table>

5. Cross Hedging

Although there is no direct hedging instrument for firm 2, cross hedging opportunity may be present. Suppose that a futures contract between the currencies in countries C and D serve this purpose. Assume that at time 0 by selling a unit of the futures contract, firm 2 agrees to exchange a unit of country C’s currency for \( k \), of country D’s currency at time 1. Let \( z \) denote the units of futures contract firm 2 sells. The profit at time 1, should it win the bid, is:
where a unit of country C’s currency is exchanged for \(e_{i_1}\) units of country D’s currency at time 1 and a unit of country D’s currency is exchanged for \(d_1\) units of country A’s currency at time 1. Otherwise, the profit is:

\[
W_{z_2} = (k_0 - e_{i_1})d_1z.
\]

As a result, firm 2 will choose \(B_2\) and \(z\) to maximize the expected utility function:

\[
EU_2 = Eu_2\left[ B_2e_{i_1} - C_2 + z(k_0 - e_{i_1})d_1\right][1 - P(B_1, B_2)]
+ Eu_{z_2}\left[ z(k_0 - e_{i_1})d_1\right] P(B_1, B_2).
\]

Assume the futures market is unbiased. Then \(k_0 = E(e_{i_1}). Let \(\eta = (k_0 - e_{i_1})d_1\). We have \(E(\eta) = -Cov(e_{i_1}, d_1)\), which is likely to be non-zero. Also note that \(e_{i_1}d_1 = e_{i_1}\). Following the mean-variance approach, we obtain:

\[
EU_2 = \left[ B_2 - C_2 + zE(\eta) - \frac{\lambda_2}{2} Var(B_2e_{i_1} + z\eta) \right] \left( \frac{B_2}{B_1 + B_2} \right)
+ \left[ -\frac{\lambda_2}{2} Var(z\eta) \right] \left( \frac{B_2}{B_1 + B_2} \right).
\]

Upon setting the partial derivative with respect to \(z\) to zero and carrying out some algebraic manipulations, we obtain:

\[
z^* = \frac{E(\eta)B_2 + \frac{\lambda_2}{2} B_2 B_1 [Var(e_{i_1}) - 2Cov(e_{i_1}, k_0d_1)]}{\frac{\lambda_2}{2} (B_1 + B_2) Var(e_{i_1} - k_0d_1)}.
\]

Under direct hedging, \(d_1 = 1\) and (32) reduces to \(z^* = B_2B_1/(B_1 + B_2)\). On the other hand, the first-order condition for the optimal bid is:

\[
\left[ B_2 - C_2 + zE(\eta) \right] \left( \frac{-B_2}{(B_1 + B_2)}^2 \right)
+ \left( \frac{B_2}{B_1 + B_2} \right) \left[ 1 - \frac{\lambda_2}{2} (B_2 - z) Var(e_{i_1} + zCov(e_{i_1}, k_0d_1)) \right]
+ \left( \frac{\lambda_2}{2} \right) \left( \frac{B_2}{B_1 + B_2} \right) \left[ (B_2^2 - 2zB_1) Var(e_{i_1}) + 2zB_1Cov(e_{i_1}, k_0d_1) \right] = 0.
\]

Letting \(d_1 = 1\), (33) reduces to (17).

In order to find the Nash equilibrium, we need to solve (22), (23), (32), and (33) simultaneously. An analytic approach is complicated. Intuitively, as cross hedging
provides benefits over no hedging, we expect the effect of cross hedging to lie between the effects in the two cases studied in the previous two sections.

6. Concluding Remarks

This study provides a comparative analysis of the role that currency futures markets play in optimal offshore biddings. In an interactive model where two bidders located in different countries bid on a project in a third country under exchange rate uncertainty, the optimal bids decrease with increasing risk aversion. Although each firm prefers to have a larger bid (which leads to a larger expected profit), the greater currency fluctuations associated with a larger bid (should the firm win the bid) are undesirable. Futures markets provide hedging opportunities to reduce currency risk, thus allowing each firm to bid more aggressively. If both bidders have direct hedging vehicles, the effect of futures trading on the optimal bid in our setting is equivalent to a one-third reduction in the risk aversion coefficient. Both firms bid higher and achieve higher expected utility.

If only one bidder has a hedging opportunity, this bidder will present a larger bid than the other. We provide analytic results to characterize the upper and lower bounds on the optimal bids by the two firms. However, we rely upon numerical methods to conduct comparative static analysis. The results are consistent with our conjectures. That is, we find that both bidders will gain through an increase in their expected utilities compared to the case of no hedging opportunities.

References


